ELEMENTARY GRAPHIC STATICS

J. T. WIGHT.


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PREFACE

In lecturing on the subject of Graphic Statics to first-year students, the Author has always felt the want of a suitable text-book which could be recommended to students in search of a working knowledge of the application of graphical methods to the solution of the simpler problems met with in Engineering and Building Construction practice.

It is not given to every one to be mathematically brilliant, but this is no reason why the young engineer should be denied the privilege of studying much in engineering which interests and appeals to him. In the solution of engineering problems, the science of Graphics presents a ready means of circumventing the many intricate and cumbrous mathematical equations which can, all too easily, clothe comparatively simple problems with an air of mystery and difficulty.

There are those who scoff at graphical methods, forgetting that there are hundreds of first-class practical engineers who are daily solving some of the most useful problems of everyday life by such methods. The work of such men is alone a sufficient

justification of the existence of graphical methods in their application to engineering problems.

It only lies with the student to realize that every problem set out in the following pages must be carefully worked out and thoroughly understood. Mere reading of the subject matter is worse than useless.

The student who conscientiously works through all the problems and examples which follow, will have acquired much that will stand him in good stead in the performance of useful engineering work.

JOHN T. WIGHT.

HERIOT-WATT COLLEGE,
EDINBURGH, *October* 1913.

ELEMENTARY GRAPHIC STATICS

CHAPTER I

INTRODUCTION

IT is unnecessary to tell those who are interested in any branch of science, that scientific discovery depends, to no small extent, on measurement—accurate measurement. Scientific discovery is indeed restricted to a few, yet it does not necessarily follow that the need for accurate measurement is likewise restricted; the importance of it is exemplified in practically everything around us, and in no other profession, perhaps, is its importance and utility so significant as in that of engineering. In every branch of engineering its importance is undisputed, and its application, in conjunction with geometric methods, to the solution of engineering problems, provides a study of extreme utility and interest.

Accuracy, both in drawing and in measurement, cannot be too strongly emphasized, for this is the keynote of success in the solution of all problems in Graphic Statics.

The one objection levelled against this science

is the liability to error introduced by inaccurate drawing and measurement. In its application to engineering problems such an objection is hardly valid, for, by exercising moderate care in the different operations involved, we can attain to an accuracy of 1 per cent.—accuracy sufficient for all practical purposes.

Its advantages are manifold, enabling us to solve, readily and quickly, many problems which would otherwise require much intricate and cumbrous mathematical investigation. In many engineering problems we deal with forces and velocities, quantities, in themselves, purely abstract, and surely it is a great advantage to the practical mind to be able to represent such things on paper, if even in a comparative way, greater still to be able, by the deft use of a few simple drawing instruments, to discover the effects produced by such forces and velocities at any point in a given system.

Definition and Specification of a Force.—Before proceeding to treat the more graphical part of the subject, it might be well to deal briefly with "*force and its specification.*"

Force.—*Force is any cause which produces or tends to produce motion or change of motion in a body.*

This definition is quite general in its application, conveying to us only a very vague notion of a force through the medium of the effect produced thereby, and before we can put anything on paper regarding it, we must have fuller information on a few points concerning the force.

The question naturally arises: "What are the points necessary to completely specify a force?" Such a specification must embody four things, viz.:—

(*a*) *Point of Application.*

(*b*) *Direction.*

(*c*) *Magnitude.*

(*d*) *Sense.*

Graphic Representation of a Force.—Let us now consider how each of these items can be represented graphically.

Point of Application.—In all our calculations in statics we have considered forces as acting at points; if we must be accurate, how then can we represent the point of application, since our idea of a point precludes any idea of area, and any mark we may make on paper must, of necessity, cover some area, no matter how small? For our purpose we must give up this mathematical idea of a point in this connection, and consider the point of application as a small area to which we have given the name "point."

Take for illustration the case of an engine pushing a train. The points of application, in this instance, are the buffers, in themselves presenting an appreciable area, yet so small in comparison to the whole area of the end of the train as to justify our calling them "points of application." Generally speaking, the point of application is the exact region in which the force acts, so that the demands of accuracy will be satisfied if we represent the point of application by a pencil dot on the paper.

Direction.—The next point to be considered is the direction of the force. By referring to Fig. 1 we see at a glance that the force might have any number of directions, for we could imagine the line *oa* to rotate about *o* and, in each new position of *oa*, we would have a new direction of our force.

It does not, however, make our ideas of direction any more clear to say that the force makes an angle α, unless we have some predetermined base or datum line from which to make our measurements. From *o* in Fig. 1, we draw to the right a horizontal line *ox*, and from this line we make all our measurements, care being taken to measure always in the one direction, the direction adopted being contrary to that of the rotation of the hands of a clock or counterclockwise.

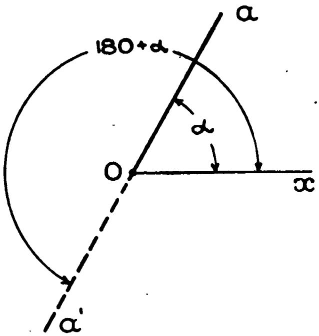


Fig. 1.

Magnitude.—The magnitude of the force is the point which next demands our attention. Anything that can be measured can be represented by a straight line drawn to some suitable scale. It is quite immaterial what length we make the line, provided we state the unit or scale to which it is to be measured. The absolute length of the line is immaterial, provided that the length of the line, the

quantity that it represents and the scale of measurement are all in harmony.

For example, say we wish to represent a force of 50 lbs.; it does not matter whether the line be 3 in. long or 4 in., if the scale be chosen accordingly, but, if our scale be fixed at 1 in.=20 lbs., then the length of the line must be 2·5 inches, and not a fraction more or less.

The choice of a scale depends on the magnitude of the forces with which we are dealing, and also on the space at our disposal on the drawing-paper. The important point to keep in mind is, that the maximum possible extent of our error in drawing is a constant, while the size of our drawings varies, so that the larger the scale, the percentage error is the less, or, in other words, the larger the scale the greater the accuracy.

Sense.—The only point remaining to be settled is the sense of the force. We may have settled and represented the three preceding points, but we may still be uncertain as to whether the force acts away from the point or towards it—whether it is a pull or a push—but this can be indicated by putting an arrowhead on the line of action of the force.

In dealing, however, with the sense of a force, it is conventional to assume that *positive forces* act *away from* the point, while *negative forces* act *towards* it, so that in all cases, unless the force be preceded by a minus sign, we take it that the force acts away from the point. Thus, for example, if we had a force of 60 lbs., pulling on a point at an angle of 45°, it would

be written, 60_{45} lbs., but if the same force were pushing on the point it would be written, -60_{45} lbs.

It may be worth noting, in passing, that $P_a = -P_{180+a}$. This will be more readily understood by reference to Fig. 1. As far as the point *o* is concerned, it is immaterial whether the force P acts along *oa* as a pull or along *oa′* as a push, provided *oa* and *oa′* are in the same straight line. Now *oa* is a positive force acting at an angle α, and hence we write it oa_α, while the force along *oa′* is a negative force, and hence we write it $-oa'_{180+a}$. But we have seen that the effect on *o* produced by either force is the same, so that $oa_\alpha = -oa'_{180+a}$. In more general terms this may be written—

$$P_a = -P_{180+a}.$$

To show the application of the ideas embodied in the foregoing definitions we will now consider the graphical representation of the force, -60_{35} lbs., acting on a given point *o*.

We are given the point of application *o* as shown in Fig. 2 (*a*), and, as for the direction, we see that it makes an angle of 35°, so that we mark off this angle, measuring counterclockwise from our datum line, *ox*. Before we can fix the length of *oa* we must select a scale, and assuming this to be 1 in. = 30 lbs., we proceed to mark off a length *oa* equal to 2 in. We further notice that the force is negative, and consequently we must put on an arrowhead acting towards the point *o*.

This diagram now represents one definite force and only one, and one advantage of graphics becomes

apparent when we compare this simple diagram with the amount of writing which would be necessary to make clear our idea of this one particular force. Note carefully that the scale of the diagram must be given alongside, otherwise the diagram is useless. It should, however, be noticed that in setting out forces in a position diagram (with which we shall deal later) it is not usual to make the length *oa* to

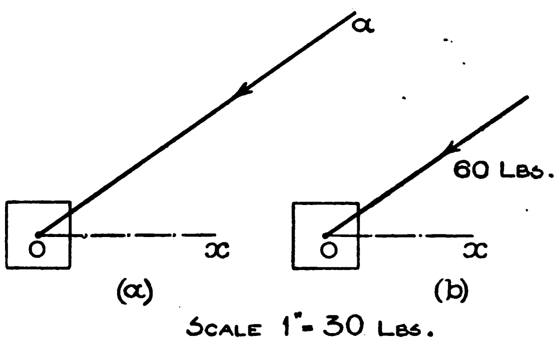


Fig. 2.

scale; the conventional method employed is to mark alongside the direction line the value of the force as shown in Fig. 2 (*b*), instead of making *oa* a definite length to scale.

Coplanar Forces.—The forces in a system may radiate in all directions from the point of application, or they may radiate in such a way that the several forces all lie in the one plane. These latter are known as *coplanar forces*, and it is intended to deal almost entirely with such in the following pages.

Concurrent and Non-concurrent Forces.—If, in any system, the several forces have a common point of application, such forces are said to be *concurrent;* but if they do not have a common point of application, but are applied to the body at various points, so situated that the several lines of action, if produced, would not intersect in a common point, such forces are said to be *non-concurrent.*

Composition and Resolution of Forces.—When we have a number of forces acting on a body we can, by suitable constructions, combine these forces, and so find one single force which, acting alone, would produce the same effect as the several forces acting together. Such a process is known as the *composition of forces*, and the single force, so found, as the *resultant.*

Resultant.—*The resultant of any system of forces is that single force which, acting alone, would produce the same effect as the several primary forces acting together.*

When we have a single force acting on a body we can, by suitable constructions, find the values of any number of forces whose combined effects, when acting in unison, produce the same effect as that of the single force acting alone. Such a process is known as the *resolution of forces*, and the several forces, so found, as *components.* It should, however, be noted, that the resolution of a force into any number of components is capable of an infinite number of solutions, but, in general, definiteness is given to the solution by practical considerations—

either the magnitudes or directions of certain of the forces will be given.

Equilibrant.—*The equilibrant of a system of forces is that single force which, when applied to that system, equilibrates or neutralizes its effect.*

The *equilibrant* must be carefully distinguished from the *resultant.* Referring to Fig. 3, and comparing these two along the lines of the specification we have drawn up, we find—

(*a*) The point of application is the same.

(*b*) The magnitudes are equal, $oa = oa'$.

(*c*) The sense is the same, both being positive and acting away from the point.

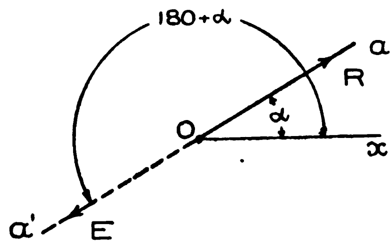


Fig. 3.

(*d*) The directions are diametrically opposed, the angle of the resultant being α, while that of the equilibrant is $180 + \alpha$.

The only point of difference then is in the directions, the two being diametrically opposed to one another, so that the effect of the equilibrant is to neutralize that of the resultant, and so bring the whole system into equilibrium. In general, when we have a given force-effect, produced by a system of forces, the resultant represents that force-effect, while the equilibrant represents that single force which must

be applied to the system to neutralize that force effect.

Units.—In absolute measurements the unit of force which is used is the *poundal*. In engineering problems, however, the unit of force is the *pound*. The poundal may be defined as that force which, acting on a mass of 1 pound, would produce in it an acceleration of 1 foot per second per second. We know that, if we drop a mass of 1 pound at the earth's surface, the acceleration produced is approximately equal to 32 feet per second per second, and hence it follows that, if the poundal produces unit acceleration, the force acting on the mass must be 32 poundals.

It is this attractive force on a mass pound that we use as our unit of force in engineering problems.

This value, however, varies slightly at different parts of the earth's surface, but the variation is so small as to be negligible in all practical problems.

The ton, consisting of 2240 lbs., is frequently used when dealing with problems involving large forces.

CHAPTER II

COMPOSITION AND RESOLUTION OF FORCES

Forces Acting in the same Straight Line.—In the case shown in Fig. 4 the method of solution should be apparent at a glance. No advantage is gained by attempting a graphical solution of such a case. The resultant is obviously the algebraic sum of the six

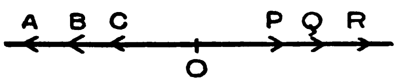


Fig. 4.

forces, and acts either to right or left, according as $(P+Q+R)$ or $(A+B+C)$ is the greater.

Resultant of Two Concurrent Forces.—Possibly the simplest and, at the same time, the most important case with which we have to deal is that in which we have to find the resultant of two concurrent forces.

Such a problem admits of two methods of solution: the first with the aid of the *Parallelogram of Forces*, the second with the aid of *position and force diagrams*.

First Method.—In solving the problem by this method, we make use of a well-known theorem, the Parallelogram of Forces. This method of solution is

not in general use in the solution of practical problems, yet there are many problems in which it can be very readily applied, and it may be worth while considering it a little in detail.

Parallelogram of Forces.—*If two forces acting on a point be represented in magnitude and direction by the adjacent sides of a parallelogram, then the diagonal through their intersection represents the resultant in magnitude and direction.*

In Fig. 5 we have two forces, P and Q, acting at a

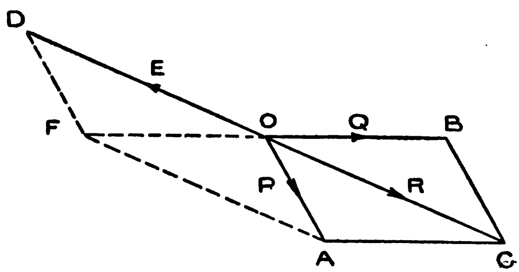


Fig. 5.

point O. Mark off along these two lines distances OA and OB equal to P and Q respectively, to some scale. Complete the parallelogram OBCA and join OC.

Then OC represents to scale the magnitude and direction of the resultant.

This important proposition was first enunciated by Sir Isaac Newton about the year 1687, and since that time various proofs have been given by different mathematicians. The following proof, due to Duchayla, is probably the best known.

Let a force P act on a body at A along the direction AB, and two forces Q and W also act on A along the direction ACD. Assume force W to act at the point C and be represented by CD to scale. Complete the parallelograms ACEB and ECDF. (Fig. 5A.)

The resultant of P and Q is assumed to be some force (say T) acting along the line AE. These two forces can be replaced by T and the point of application of the force assumed at any point in its line of

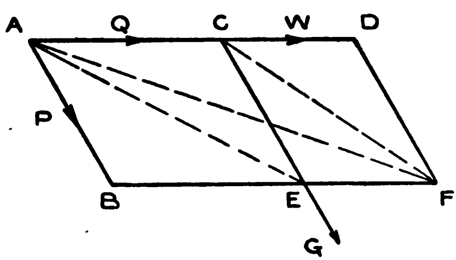


Fig. 5A.

action. Let its point of application be E. This force T, acting at E, may now be replaced by forces equal to P and Q, acting in the directions EG and EF, and further let their points of application be removed to C and F. Acting now at C, we have two forces—P acting along CE and W acting along CD. Again we can assume the resultant of P and W to be some force (say S) acting along the line CF, and by taking its point of application as F, we thereby apply all the forces at one point without altering their combined effect. Hence F is a point on the line of action of

the resultant. Therefore AF is the direction of the resultant.

For P we may write $(m \times f)$ and, for (W+Q), $(n \times f)$ and the same theorem holds. Hence in the general case the diagonal represents the direction or line of action of the resultant.

We must now consider the magnitude of the resultant. Referring back to Fig. 5, we apply to the system a force E, represented by OD, equal and opposite to the combined effects of P and Q.

We know that the resultant of P and Q acts along OC, hence OD and OC will be in the same straight line, and if OC represents the magnitude of the resultant, then OC must equal OD. Complete the parallelogram ODFA, and join OF. Now the resultant of E and P will act along OF, and since the point O is in equilibrium under the action of three forces P, Q and E, it follows that OB and OF must be in the same straight line. The figure OFAC is therefore a parallelogram, and consequently FA equals OC. But FA equals DO, hence OC equals OD.

The length OC therefore represents the magnitude of the resultant of P and Q, the inclination of the line representing its direction.

This theorem can be demonstrated quite simply by the aid of two spring balances and a weight of known magnitude, the apparatus being set up as shown in Fig. 6. A and B are two fixed points to which two spring balances are attached. A length of cord connects the hooks of the balances, and at the point O, in this cord, is hung a weight of known

magnitude W. In the position of equilibrium the three forces will act along the cords OA, OB and OW as shown. It is a simple matter to transfer the position of the cords to a sheet of paper, when the magnitude of s_1 can be set out as O*a*, and s_2 as O*b*. Draw *ac* parallel to O*b* and *bc* parallel to O*a*. Join O*c*. It will now be noted that O*c*, when measured, equals W, and is a direct continuation of the line OW.

Second Method.—The method now to be adopted is worthy of careful attention, as the principles involved underlie the graphical solutions of practically all engineering problems dealing with forces.

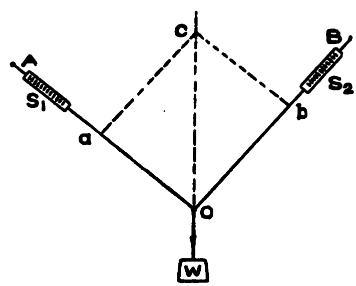
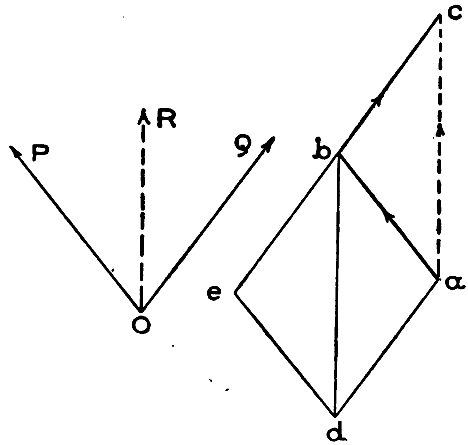


Fig. 6.

We here make use of two different figures, known respectively as the *position diagram* and the *force diagram*, as shown in Fig. 7.

In setting out our forces we saw that we measured our angles anticlockwise from the datum line, but, in transferring our forces from the position diagram to the force diagram, it is found to be more convenient to go always clockwise round the point, taking the forces in rotation. It does not matter how we go round the point or in what order we take the forces, but it is advisable to cultivate some system and orderliness in dealing with such problems, and the above system is the one which answers the purpose best.

We start by drawing a line *ab* parallel to the force P, and of such a length that *ab* represents to scale the magnitude of P, the direction of lettering *a* to *b* being in accordance with the sense of the force; from *b* we next draw a line *bc* parallel and equal to Q. Join *ac*, and through O in the position diagram we draw a line R parallel to *ac*.



Position Diagram. Fig. 7. Force Diagram.

This line *ac* now represents the magnitude of the resultant, while the line OR represents its direction, the sense being indicated by the arrowhead.

To show that this is actually the case, we will now consider the resultant from the following three points of view—

(*a*) Magnitude. (*b*) Direction. (*c*) Sense.

Dealing first with the magnitude, we will show that *ac* represents the value of R. Produce *cb* to *e*, making *be* equal to *cb*, complete the parallelogram *abed* and join *bd*. Considering now the point *b*, it does not matter whether the force *bc* acts as a pull from *b* to *c*, or as a push along *eb* from *e* to *b*, since the two forces are equal in magnitude ($P_a = -P_{180+a}$) and hence the resultant of *ab* and *eb* will be the same as the resultant of *ab* and *bc*.

In the previous proof we showed that the diagonal *bd* equalled the resultant in magnitude and direction, and therefore, if *ac* is the resultant of *ab* and *bc*, then it must be parallel and equal to *bd*.

Compare the triangles *cba* and *bed*.

cb is equal and parallel to *eb*,
ba is equal and parallel to *ed*,
Angle *abc* is equal to the angle *deb*.

Hence the triangles are equal in every respect, and *bd* is therefore parallel and equal to *ac*.

Therefore *ac* represents the resultant in magnitude and direction.

It is not advisable to put arrowheads on the force diagram, but they have been added in this case to show the method of investigating the sense of the force. We know that the resultant must act as at R, which is from *a* to *c* in the force diagram, and it will be noticed that this direction of the arrowhead is the opposite way round to the other forces *ab* and *bc*. It is important to note that this rule always holds good.

Rule.—*In the force diagram the resultant always*

points the opposite way round to the other forces, while the equilibrant points the same way round.

The Triangle of Forces.—*If three forces acting on a point be represented in magnitude and direction by the three sides of a triangle taken in order, then these three forces are in equilibrium.*

The meaning of this theorem will be better understood by a reference to Fig. 8. Here we have three forces, P, Q and S, acting on a point O in equilibrium.

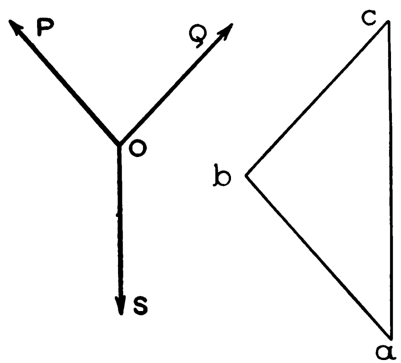


Fig. 8.

To construct our force diagram we begin by selecting one of the forces, say P, and drawing a line *ab* parallel and equal to it; from *b* we draw a line *bc* parallel and equal to Q, and finally a line *ca* parallel and equal to S.

Now, if the three forces are in equilibrium, the end *a* of the line *ca* will coincide with the end *a* of the line *ab*, for, if they did not coincide, we would require a closing line for our force diagram, which is an evidence of the existence of a resultant, and such, we know from the hypothesis, does not exist, the forces being in equilibrium.

The triangle *abc* represents the magnitudes and directions of the three forces, P, Q and S.

The phrase *taken in order* means that the arrowheads indicating the sense of the forces follow each other in sequence round the sides of the triangle, and it is worth noting that the direction of lettering, which is coincident with the sense of the forces, also follows round the triangle in order.

It is important to notice the fact that, if three forces act on a body in equilibrium, their directions must be concurrent. Considering P and Q, we know that their resultant must pass through O, and since the forces are in equilibrium the force S must form the equilibrant of P and Q, and must also pass through O.

Any one force is the equilibrant of the other two. The foregoing theorem is, however, only a particular case of a more general theorem, known as—

The Polygon of Forces.—*If any number of forces, acting on a point, be represented in magnitude and direction by the sides of a polygon taken in order, then these forces are in equilibrium; if the polygon does not close, then the closing line drawn from the starting to the stopping point represents in magnitude and direction the resultant of that system of forces.*

Consider the case shown in Fig. 9, where we have five forces, P, Q, S, T, W, acting at a point O, assuming that we are required to find the resultant.

Starting then with the force P, we commence our force diagram by drawing a line *ab* parallel and equal to P. The next force in rotation is Q, and from *b* we now draw a line *bc* parallel and equal to Q, and so on with each of the several forces until W is reached in the line *ef*.

Should the end f of the line ef coincide with the end a of the line ab, we would then have a closed figure, and the five forces, P, Q, S, T, W, would be in equilibrium.

If, however, f does not fall on a, we join af and through O in our position diagram draw a line parallel to af.

This line now represents the line of action of the

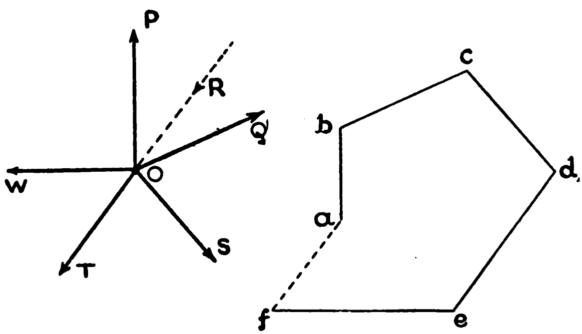


Fig. 9.

resultant R, while the length af represents its magnitude, its sense being found by following the forces round the force diagram.

Note.—It is immaterial in what order the forces are taken, as can be easily proved by drawing out several polygons and comparing results, but, although this is the case, it is advisable to cultivate some system of orderliness in tackling such problems; the habit thus formed will obviate much worry and confusion when dealing with more intricate problems.

Resolution of Forces.—The two general cases just given provide a means of solving all problems involving the composition of concurrent forces. The various operations are simply reversed in the application of these constructions to problems dealing with the *resolution of forces.*

It is hardly necessary to deal very fully with this part of the subject, as the restrictions imposed on the solutions reduce the problems, in general, to exercises in simple geometry.

In general, whether resolving into two forces or any number of forces, we are reduced to four particular cases. We have to find one or other of the following:—

(*a*) Direction and magnitude of one force.

(*b*) Direction of one and magnitude of the other.

(*c*) Directions of two forces.

(*d*) Magnitudes of two forces.

This can be expressed generally thus: If n be the number of forces acting at a point, then to specify the forces at the point we must know $2n$ facts about the forces, *e.g.*, n magnitudes and n directions. It is only possible to solve the problem if $(2n-2)$ facts are known about the given forces.

Probably the method of dealing with such problems will be most easily understood by an examination of a few particular cases worked out to scale and dealing with all the cases mentioned in the previous pages.

Examples worked out.

1. A weight of 500 lbs. is hung by two cords, *oy* and *oz*, which make angles of 50° and 150° respectively,

with the datum line as shown. Find the tension in each of the cords. Scale 1 in. = 250 lbs. (Fig. 10.)

Produce the line of action of the load and mark off *ob* equal to 2 in. From *b* draw *bc* parallel to *oz* and *ba* parallel to *oy*.

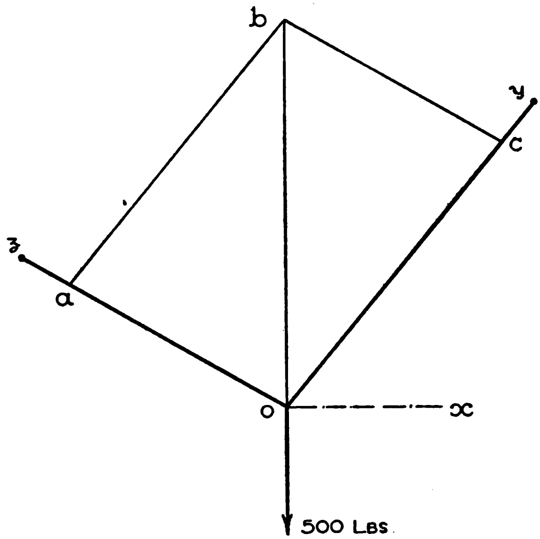


Fig. 10.

Then *oc* equals the tension in the cord, *oy* and *oa* equals the tension in the cord *oz*.

$$oc = 1{\cdot}75 \text{ in.} = 1{\cdot}75 \times 250 = 437{\cdot}5 \text{ lbs.}$$

$$oa = 1{\cdot}32 \text{ in.} = 1{\cdot}32 \times 250 = 330{\cdot}0 \text{ lbs.}$$

2. Two forces are applied to a point O, whose resistance to motion is 400 lbs., acting at 270°. If the directions of the applied forces be 60° and 135°

respectively, find the value of each force when motion just takes place? Scale 1 in. = 200 lbs. (Fig. 11.)

When motion just takes place, the resistance of 400 lbs. has just been overcome, so that the point O is in equilibrium under three forces, viz.: A resistance of 400 lbs., acting away from O, and two forces P and Q, acting away in the opposite directions, as shown.

We begin by drawing a line *ab*, 2 in. long, parallel to R; from *b* a line *bc* parallel to P, and from *a* a line parallel to Q; *acb* now represents the triangle of forces for the point O, and *bc* and *ca* represent the forces P and Q respectively.

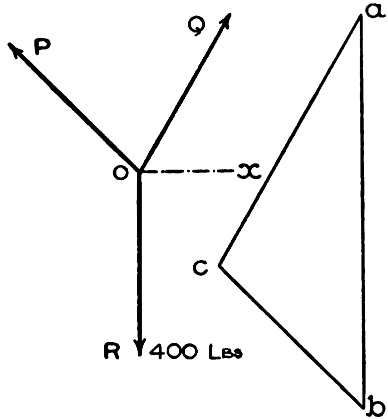


Fig. 11.

$$bc = 1{\cdot}05 \text{ in.} = 1{\cdot}05 \times 200 = 210 \text{ lbs.}$$
$$ac = 1{\cdot}45 \text{ in.} = 1{\cdot}45 \times 200 = 290 \text{ lbs.}$$

3. Five men pull on a body as shown, each with a force of 30 lbs. It is proposed to apply, by means of a tackle, a single force, to have the same effect as that of the five men. Find the position of the tackle and the force which it must be capable of exerting. Scale 1 in. = 60 lbs. (Fig. 12.)

We begin by drawing a line *ab*, ½ in. long, parallel

to P_1, then *bc* parallel to P_2, and so on until P_5 is reached in the line *ef*. Join *af*, and *af* represents the magnitude of the pull exerted by the tackle, while its direction, as represented by R, indicates the centre-line.

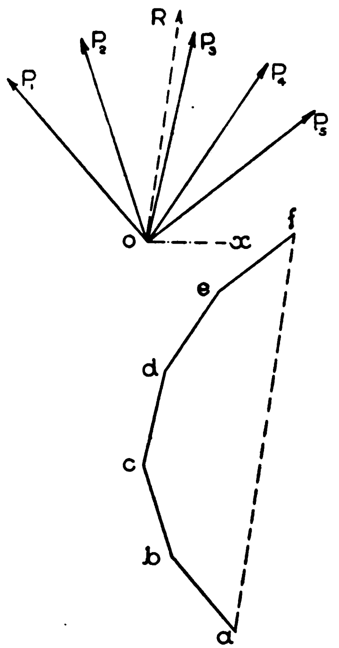


Fig. 12.

$af = 2{\cdot}09\text{in.} = 2{\cdot}09 \times 60 = 125$ lbs.

Direction of $R = 83°$.

4. A derrick pole is guyed by means of five tension rods, four of which, P, Q, R, S, are shown in plan. It is known that the horizontal forces exerted on the top of the pole by these guys are 2, 3, $2\frac{1}{2}$ and $1\frac{1}{2}$ tons respectively. (Fig. 13.)

Find the direction and magnitude of the horizontal force exerted by the fifth guy. Scale 1 in.$=$2 tons.

Beginning with P, we draw a line *ab*, 1 in. long, parallel to P, and so on with the other forces until S is reached in the line *de*. Join *ea*, and through O draw a line T parallel to *ea*. Then *ea* represents the horizontal force exerted by the fifth guy, while T represents its direction.

$ea = 1{\cdot}38$ in. $= 1{\cdot}38 \times 2 = 2{\cdot}76$ tons.

Direction of guy $= 301°$.

5. The point O is the plan of a telegraph pole, from which six lines of wire radiate. The positions

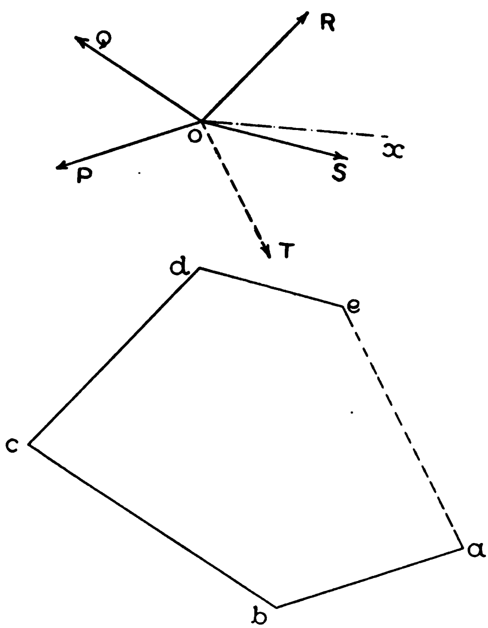


Fig. 13.

of four of them are indicated by the lines, P, Q, R, S, the tensions in each being 20 lbs. It is known, regarding the other two lines, that one of them is inclined at 180°, while the other is in tension to the extent of 30 lbs. If O be in equilibrium, determine

the full particulars of the latter two lines. Scale 1 in. =20 lbs. (Fig. 14.)

Beginning, as before, with P, we draw a line *ab* 1 in. long, parallel to P, and so on with each force until T is reached in the line *ey* (of indefinite length).

The polygon is still unclosed, but we have yet to add to it a force of 30 lbs. of unknown direction,

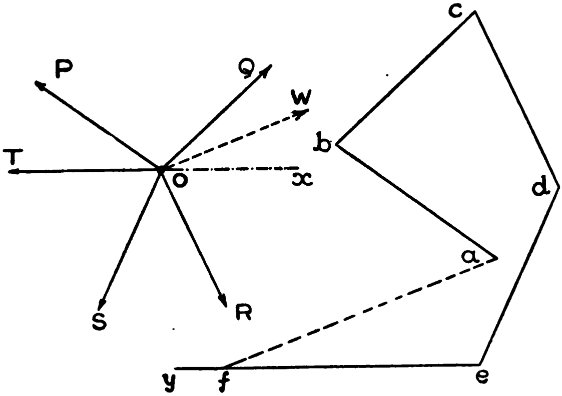


Fig. 14.

and this force must close the polygon. With centre *a* and radius *af*, equal to 1·5 in., cut *ey* in the point *f*.

Then the two lines *ef* and *fa* give us the required information regarding the two unknown lines:—

$$ef = 1{\cdot}30 \text{ in.} = 1{\cdot}3 \times 20 = 26 \text{ lbs.}$$

$$\text{Direction of } af = 22^\circ.$$

$$\therefore\ T = 26_{180^\circ} \text{ lbs. and } W = 30_{22^\circ} \text{ lbs.}$$

6. A barge, stuck in a canal bank, is to be hauled

off by means of two winches placed on the opposite bank. It requires a pull of 1000 lbs. to just pull the barge off, and the winches are capable of exerting pulls of 600 and 900 lbs. respectively. Find the limiting directions of the hauling ropes relative to a line drawn perpendicular to the bank through the point at which the barge is stuck. Scale 1 in.= 500 lbs. (Fig. 15.)

Assume O to be the point at which the barge is stuck. Through O draw a perpendicular O*x*. The resistance of the barge is assumed to act along this line, and therefore we draw, parallel to O*x*, a line *ab* 2·0 in. long.

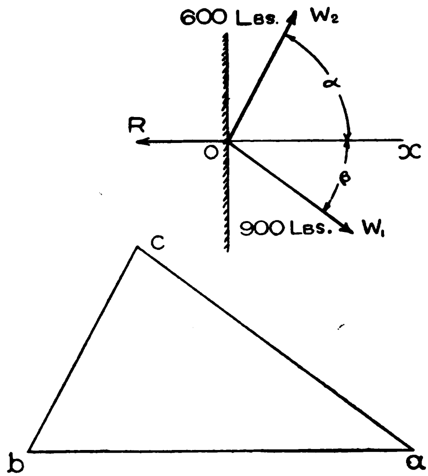


Fig. 15.

With centre *b* and a radius of 1·8 in. (representing the pull of the 900-lb. winch) describe an arc, then with centre *a* and a radius of 1·2 in. (representing the pull of the 600-lb. winch) cut the arc at *c*.

Join *ac* and *bc*, and through O draw OW_1 and OW_2 parallel to *ac* and *bc* respectively.

These lines now represent the directions of the ropes—

Rope of small winch at $61° = \alpha$.

Rope of large winch at $36° = \beta$.

7. If, in the previous question, the positions of the winches were such that the ropes were inclined to Ox at angles of 30° and 50° respectively, what would have been the minimum power of the winches necessary to give a pull of 1000 lbs. on the barge? Scale 1 in. = 500 lbs.

Set out a line ab, 2·0 in. long, parallel to Ox; through b draw bc parallel to W_1, and through a draw ac parallel to W_2.

Then ac and bc represent the pulls exerted by the winches W_2 and W_1 respectively.

$$ac = 1{\cdot}05 \text{ in.} = 1{\cdot}05 \times 500 = 525 \text{ lbs.}$$

$$bc = 1{\cdot}57 \text{ in.} = 1{\cdot}57 \times 500 = 785 \text{ lbs.}$$

Therefore winches must be capable of exerting pulls of 785 and 525 lbs. respectively.

Bow's Notation.—In nearly all our problems we deal with two kinds of diagrams, viz., *position diagrams and force diagrams.*

In the *position* diagram, which is drawn to some *space scale*, the lines only indicate the *directions of the forces*, while, on the other hand, in the *force* diagram, which is drawn to some *force scale*, the lines represent the *magnitudes of the forces* acting at the corresponding points in the position diagram.

In passing thus from one diagram to the other, it is essential that we have some method of lettering common to both, yet distinct in each. The method

which is probably the best is Bow's (sometimes known as Henrici's) notation. This method of lettering is extremely useful in dealing with forces in equilibrium, as in the case of the forces in the various members of a roof truss or braced girder, but it is not advisable to use it in the solution of

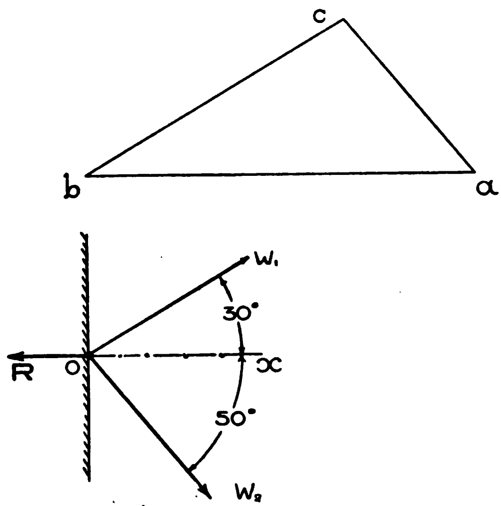


Fig. 16.

problems such as we have dealt with in the preceding pages. Possibly the best method of explaining the notation would be to consider an actual case. Consider the case of the four forces, P, Q, S and T, acting on the point O—O being in equilibrium. Instead of naming the forces, as we have done, P, Q, S, T, we put a capital letter on either side of the

line of action of the force, or we do, what amounts to the same thing, we letter the spaces between the lines of action of the forces as shown, the direction of the lettering being clockwise. Thus we call the space between T and P the space A, between P and Q the space B, and so on. Suppose now we wish to indicate the force P, we speak of it as the force AB, Q becomes the force BC, and so on.

Passing on to the force diagram, we begin by

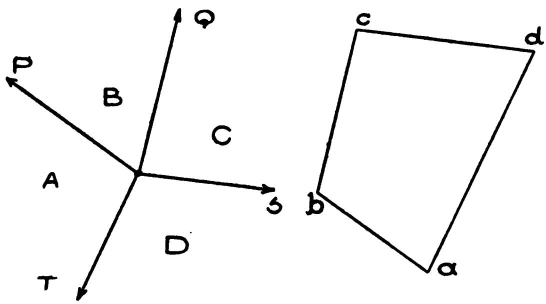


Fig. 17.

drawing a line *ab* parallel to the force AB, and the following point should be carefully noted regarding this line. The italics are used in figuring it and the force AB is represented by the line *ab*, the sense being indicated by the direction of lettering *a* to *b*; so that in our *position diagram* the forces are indicated by *capital letters*, while in our *force diagram* they are indicated by the corresponding *italics.*

The polygon is finished in the usual way, and an examination of Fig. 17 will serve to convince the

student of the extreme utility of this system of lettering.

Examples.

1. Forces of 32 lbs. and 24 lbs. act on a point in such a way that the angle between their lines of action is 73°. Determine the magnitude of the resultant force.

2. A body O is acted on by two forces, P and Q. P is equal to 90 lbs. and acts at an angle of 28° to the datum-line, while Q is equal to 72 lbs. and acts at 103°. Determine the magnitude and direction of that force which would just prevent O being moved by the forces P and Q.

3. A chain-sling, 18 ft. long, is used to lift a casting, weighing 45 tons. The ends of the sling are attached to eyebolts on the casting at horizontal points 3 ft. 6 in. apart. Determine the pull in the chain when the casting is suspended by the chain.

4. Four forces, P, Q, S, T, act on a point and keep it in equilibrium. It is known that the values of P and Q are 200 and 280 lbs. respectively, while the angles at which they act are 30° and 110° respectively. S and T act at 220° and 280° respectively. Determine the magnitudes of the latter two forces.

5. A girder, weighing 30 tons, is hanging from a crane, the length of the suspending rope being 26 ft. It is found necessary to deflect the girder horizontally through a distance of 3 ft. Determine the horizontal force necessary to accomplish this amount of horizontal deflection.

6. A barge is being pulled along a canal by means

of a tow-rope. The barge is 34 ft. from the bank while the horse is 5 ft. from the edge of the bank. The tow-rope is 64 ft. long. The horse exerts a pull of 140 lbs. Determine the resistance to motion per ton if the loaded barge weighs 25 tons.

7. A steel chimney is 20 ft. high and 30 in. diameter. It is guyed by four wire ropes running north, south, east and west. The ropes are attached to a strap 10 ft. up from the ground and slope at an angle of 34° with the ground. A north wind blows with a force of 30 lbs. per sq. ft. of projected area. Determine the pull in the north guy, assuming it taking all the load. What would be the pull in the north and east guys, with a north-easterly wind of intensity equal to 40 lbs. per sq. ft.?

CHAPTER III

PRACTICAL PROBLEMS

HAVING now discussed the principles involved in the solution of the simpler general problems in Graphic Statics, we will now proceed to consider the application of these principles to a few simple problems commonly met with in practice.

Loaded Platform.—In Fig. 18 is illustrated a uniformly loaded platform HP, which is hinged at H, and held in its horizontal position by means of two tie-rods FP, one at either end of the platform. In this case suppose we let

L = the length of the platform in ft.
B = the breadth of the platform in ft.
w = load per sq. ft. in lbs.
W = total load carried.

Then $W = L \times B \times w$ lbs.

We have assumed the load in this case to be uniformly distributed, and hence W can be taken to act through the centre of area of the platform. Bisect HP in the point K and through K raise a vertical to cut FP in the point O. Now the platform is acted on by three forces which keep it in equilibrium, viz.: the load W, the tension in the tie-rods and R, the reaction at the hinge. We have seen that when we

have a body acted on by three forces, which keep it in equilibrium, then these three forces must pass through a common point. The load W and the tension T meet in O, hence the hinge reaction must also pass through this point. By drawing a line to pass through H and O we get the line of action of the reaction. This now completes the position diagram, from which it is a simple matter to obtain the force diagram and determine the magnitude of the required forces. Set down, to scale, a line *ab* to represent the load W; from *b* draw *bc* parallel to R, and from *a* draw *ac* parallel to T. The triangle *abc* gives the force diagram from which we can scale off the forces in the tie-rods and at the hinge.

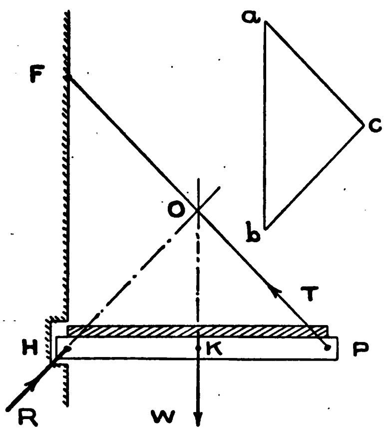


Fig. 18.

Note.—Tension in each tie-rod$=\frac{T}{2}=\frac{ac}{2}$ lbs.

Triangular Frame.—A triangular frame ABC is supported at A by a hinge and at C by a rope which passes over a pulley D. At B a load W is attached. Determine the pull in the rope and the reaction at the hinge. Produce the line of action of the rope at C until it cuts BW in O; draw a line to pass through

A and O. Produce OB and mark off O*b* to represent W, to scale. From *b* draw *ba* parallel to OC and *bc* parallel to OA. Then O*a* measured to scale gives the magnitude of the reaction at A, while O*c* gives the tension in the rope.

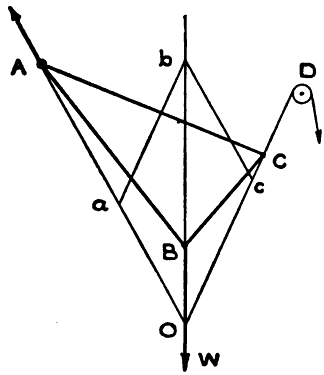


Fig. 19.

Shear Legs. — The shear legs, shown in Fig. 20, as used for lifting heavy loads, introduces one or two points of interest. The arrangement consists of two struts, as shown by the member CB, and a back-tie, as shown by AC. In the first part of the problem we make use of the triangle of forces to determine the magnitude of the pull in the tie AC, and the thrust which is sustained by the two struts. Set down *ab* to represent W and complete the triangle of forces *abc*. The line *ac* gives the pull in the back-tie, while *bc* gives the thrust sustained by the two struts. This latter thrust acts along a centre-line between the struts, and we have now to determin how this thrust is sustained. In order to see how this force *bc* acts with respect to the struts, we must first obtain an elevation looking in the direction of the arrow H. From B, the foot of the strut, raise a perpendicular; with centre B swing round the strut

BC until it cuts the perpendicular in E. Select any point F on the ground line and set out on either side half the spread of the feet of the struts in GK; project along E to cut the vertical from F in L. Join LG and LK. Then GLK is the true elevation as seen in the direction of the arrow H. The force *bc* acts along LF and is resisted by forces in LG and LK. From L set off LM equal to *bc*. From M

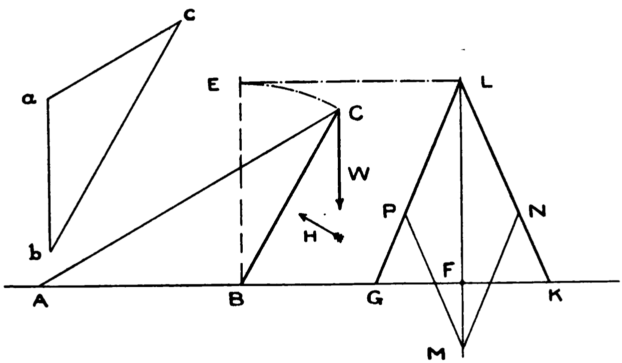


Fig. 20.

draw MN parallel to GL, and MP parallel to KL. Then LP and LN represent the thrusts in LG and LK respectively.

Tripod.—Another simple arrangement, either for lifting or sustaining loads, is shown in Fig. 21. The arrangement in this case consists of three struts, connected at their top ends, from which the load is hung. Draw the elevation of the tripod in such a way that OA shows the true length of one of the struts. The struts, being all of the same length,

make equal angles with ground and centre line and hence each one sustains a load proportional to one-third of the total load. Mark off on the centre line a length Oa equal to $\frac{W}{3}$; project ab horizontally to cut OA in b. Then Ob represents the thrust in each of the legs.

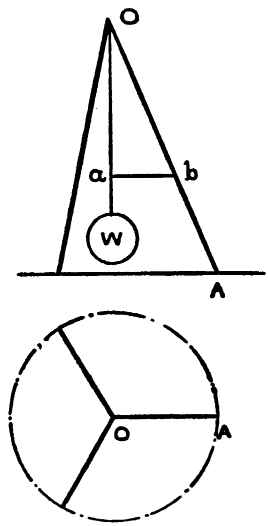


Fig. 21.

Simple Crane with Suspended Load.—Fig. 22 shows a very simple type of crane, in which the load is lifted by a tackle suspended from the outmost point of the jib. It is required to find the thrust in the jib BC and the pull in the tie AC (Bow's notation). Set down ab to represent W, from b draw bc parallel to BC, and from a draw ac parallel to AC. Then bc gives the thrust in the jib, while ac gives the pull in the tie-rod.

Simple Crane with Single Chain Lift.—Fig. 23 shows a modification of the crane in the previous question. The load, in this case, is lifted by a chain which passes over a pulley at the end of the jib, running back parallel to the tie-rod and passing over another pulley at the top of the mast. Set down ab to represent W; from b draw bc parallel to BC. We

have now two forces acting at the jib-end, namely, the pull in the tie-rod CD and the pull in the chain DA. Concerning these two forces we know the following facts—the pull in the chain has a magnitude, W, its direction is parallel to the tie-rod, and it acts away from the jib-end; while, regarding the force in the tie-rod, we only know that it acts along CD. We can therefore continue our force diagram

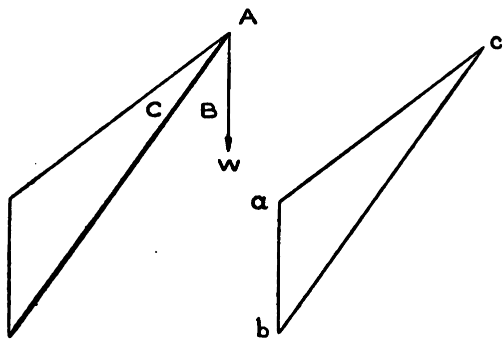


Fig. 22.

by drawing from a a line ad parallel to the chain and having a magnitude equal to W. The force diagram can now be completed by drawing from d a line parallel to DC cutting bc in the point c.

The student, in solving this problem, should use the same configuration and load as in the previous example, and note that the thrust in the jib is the same in each case, while in the second case the pull on the tie-rod is reduced by W.

Simple Crane with Double Chain Lift.—The out-

line shown in Fig. 24 shows an important modification of the lifting gear as used in the previous crane. The load W in this case is lifted through the medium of a snatch-block. The lifting rope is attached to a point at the jib-end. This rope passes down and round the pulley of the snatch-block, thence up and over the pulley at the point of the jib. It then passes over the winding barrel, which in this case is

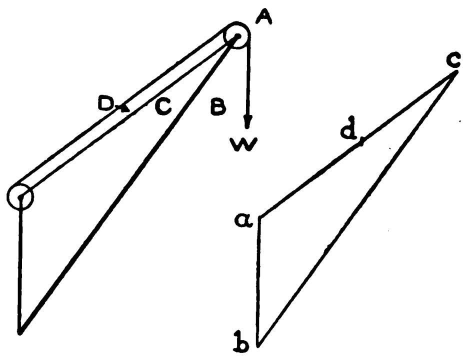


Fig. 23.

situated about the centre of the mast. Two important differences from the previous example should be noted. The load W is now supported by two ropes, and hence, neglecting friction, the pull throughout the rope will equal $\frac{W}{2}$ instead of W, as previously. Further, the lifting rope passing from the point of the jib to the winding barrel is no longer parallel to the tie-rod. In solving this problem we have two methods open to us. We can either treat each force

separately, or we can find the resultant force at the jib-point due to the magnitude and direction of the pulls in the various ropes. Taking each force separately, we first set down $ab - bc$ to represent the pulls at the jib-point caused by the two rope falls. It will readily be seen that $ab = bc = \frac{W}{2}$. From c we now draw cd parallel to CD, but we cannot, as yet, fix the

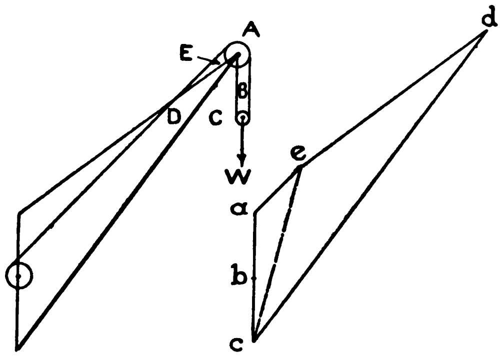


Fig. 24.

point d, since we do not know the magnitude of the stress in CD. We, however, know both the magnitude and the direction of the pull in AE, and we can therefore draw from a a line ae parallel to AE and of magnitude equal to $\frac{W}{2}$ (the constant tension in the rope). From e we can finally draw ed parallel to ED, and so complete the force diagram. The stresses in the jib and tie-rod can then be scaled off from the diagram.

When the second method is adopted we first find the magnitude of the resultant (R) of the pulls acting in the three ropes at the point of the jib. Complete the parallelogram, remembering that OM = W and ON = $\frac{W}{2}$, and find the resultant R. Set down *ab* = R, and complete the triangle of forces *abc*. By comparing the two diagrams it will be found that exactly

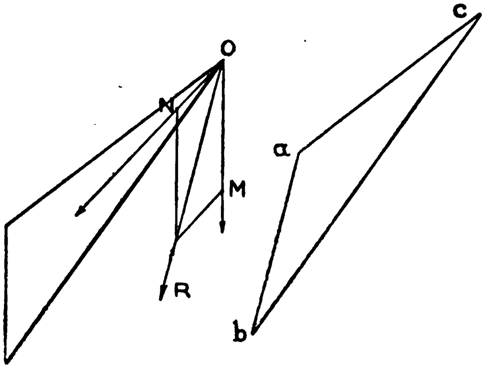


Fig. 25.

the same result is obtained in each case. If this force triangle *abc* be traced on tracing paper and superimposed on the force diagram obtained by the previous method, it will be found that the line *ab* will coincide with a line joining the points *c* and *e*, as shown dotted.

Rotating Crane.—Fig. 26 shows a simple type of crane which is mounted on a central pivot at O, and capable of rotating through a complete circle. In

order to maintain equilibrium when lifting the load W, it is necessary to apply a balance weight W_1 of such a magnitude that the moment of W_1 about O will just neutralize the moment of W about O. The solution of this problem should present no difficulty if the joints be taken in the following order—point of jib, head of mast, outer end of lower member DE.

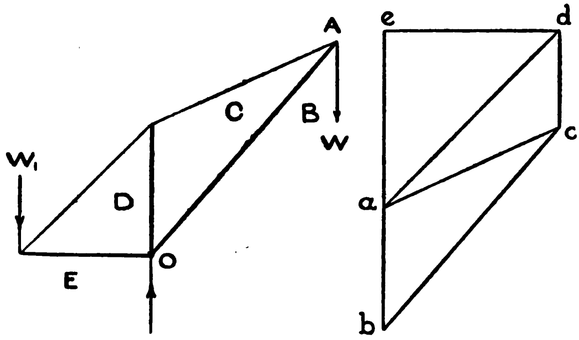


Fig. 26.

Notice that *ea* gives the magnitude of the balance weight W_1.

Warehouse Crane.—Fig. 27 shows a simple warehouse crane as fitted with a lifting tackle hung from the point of the jib. This crane is capable of swinging through a considerable angle, being mounted on a pivot bearing at the lower extremity of the post CD, and fitted with a simple bearing at the upper extremity. This upper bearing exerts only a horizontal force on the post, while the lower pivot exerts both a vertical and a horizontal force on its lower

extremity. In solving this problem we first determine the magnitude of the two reactions. We know that for equilibrium the three forces acting on the crane structure must all pass through a common point; R_1 and W intersect at O, and hence R_2, the

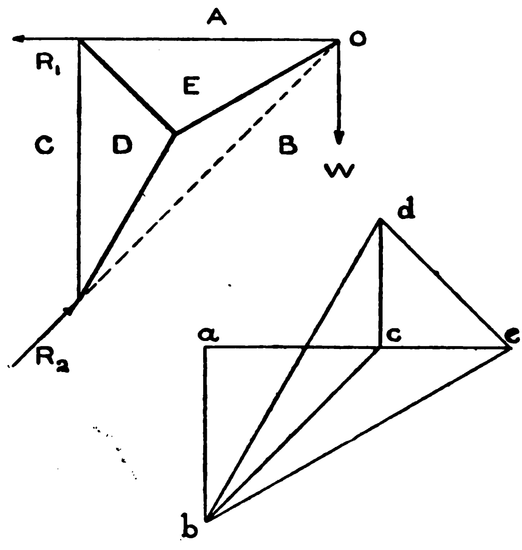


Fig. 27.

lower reaction, must also pass through this point. Knowing the three directions and the magnitude of W, we can easily find R_1 and R_2 by drawing the triangle of forces *acb*. The solution of this problem will be easily understood from a study of the completed force diagram.

Examples.

1. A draw-bridge, 30 ft. long and 20 ft. wide, is supported in a horizontal position in the manner shown in Fig. 18. The angle between the chains and the beam is 56°. Find the tension in each of the chains and the reaction at the hinge when the bridge is carrying a uniformly distributed load of 120 lbs. per sq. ft.

2. A small crane consists of a beam 10 ft. long, hinged at its inner end. The outer end is supported by a steel wire-rope, which is attached to the wall at a point 8 ft. above the hinge. A load of 4 tons is lifted by means of a tackle, attached to a point 3 ft. from the outer end. Find the reaction at the hinge and the pull in the rope.

3. A gate, 4 ft. high and 6 ft. wide, is carried on two hinges, the lower one at the bottom right-hand corner taking all the vertical load. The hinges are 3 ft. 6 in. apart. Determine the magnitudes and directions of the reactions at the hinges if the gate weighs 200 lbs.

4. A set of shear legs is used to place a boiler on its seating. The legs are each 40 ft. long and the spread of the feet 28 ft. 6 in. The plane containing the struts makes an angle of 70° with the ground line and the back tie is 54 ft. long. Find the pull in the tie and the thrust in each of the legs if the boiler weighs 16 tons.

5. A tripod, consisting of three struts, each 30 ft. long, is used to lift a steam-engine cylinder weighing 2·4 tons. The struts make equal angles with the

ground, and their lower extremities lie on a circle 12 feet diam. Find the thrust in the struts.

6. A crane similar to that shown in Fig. 22 has the following dimensions: Mast, 10 ft.; jib, 28 ft.; and tie, 21 ft. Determine the pull in the tie and the thrust in the jib when lifting a load of 4 tons.

(*a*) Simple case, as in Fig. 22.

(*b*) With chain lift, as in Fig. 23.

(*c*) With snatch-block, as in Fig. 24, the lifting barrel being 4 ft. from the foot of the mast.

7. A revolving crane, as shown in Fig. 26, has the following dimensions: Mast, 10 ft.; centre of mast to centre of load, 20 ft.; centre of mast to centre of balance weight, 12 ft.; point of jib, 18 ft. above the ground level. Determine and tabulate the stresses in the various members when lifting a load of 10 tons.

8. A warehouse crane, of the type shown in Fig. 27, is used for lifting bales up to a maximum of 30 cwts. Determine the stresses in the members if the crane has the following proportions: $AE = CD =$ 12 ft., and $DE = 7$ ft. Determine also the reactions R_1 and R_2.

CHAPTER IV

COMPOSITION OF NON-CONCURRENT FORCES

The Funicular Polygon.—So far, we have been dealing only with concurrent forces, but we shall now proceed to investigate the case in which the several forces have not a common point of application.

In dealing with concurrent forces we saw that the one condition necessary for equilibrium was that the *force polygon should close,* but, in dealing with non-concurrent forces, a further condition must be satisfied, namely, *the funicular polygon must also close.*

Before investigating the latter condition, it might be well to explain the term "funicular polygon." The word "funicular" has its derivation in a Latin word "*funis,*" meaning a "*cord,*" and the funicular or link polygon represents the shape which would be assumed by an endless cord, were it subjected to the given system of forces.

In many cases the forces acting in the various links of the funicular polygon are in the nature of compression. As is well known, a string cannot transmit a compression, and hence the general use of the term *funicular polygon* is hardly appropriate. The term is retained principally for convenience, and has now only a geometrical significance.

In dealing with this case, let us consider the general example shown in Fig. 28, in which we have four forces, P, Q, R and S, forming a system in equilibrium, the figure *pqrs* being the shape assumed by the cord under the action of the given system.

Beginning with the point *p*, we see that we have here a point in equilibrium under the action of three concurrent forces, OA, AB, and BO, and hence we can represent these three forces by the sides of a triangle, as explained in Chapter II. By treating each of the four points *p*, *q*, *r* and *s* in the same way, we get four triangles, *aob*, *boc*, *cod* and *doa*, and if we examine them carefully we find a close relationship exists among them; we find, for example, that *ob* in (*p*) is equal and parallel to *ob* in (*q*); similarly with *oc* in (*q*) and *oc* in (*r*); with *od* in (*r*) and *od* in (*s*); with *oa* in (*s*) and *oa* in (*p*).

The positions of the separate triangles are, however, quite arbitrary, and hence there is no reason why we should not place them in such positions that they will form a complete figure *abcd* as shown. There are now a few points in the two diagrams worthy of comparison. We know that the lines *ab*, *bc*, *cd* and *da* are each equal and parallel to AB, BC, CD and DA respectively, and therefore the figure *abcd* is the *force polygon* for the system. This figure forms a closed figure, thereby satisfying one condition of equilibrium, and it is worth noting that a system of *radiating lines in the position diagram* has now become, *in the force diagram, a system forming a closed figure.*

We also know that the lines *oa*, *ob*, *oc* and *od* are each equal and parallel to the forces existing in the members OA, OB, OC and OD respectively, and here again we find *a system of radiating lines*, in this

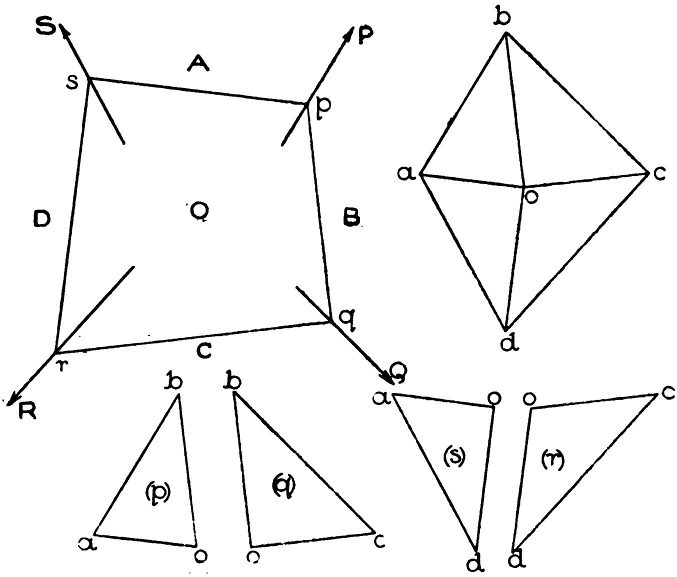


Fig. 28.

instance *in the force diagram* converted into a *system forming a closed figure in the position diagram.*

When such a relationship exists between two diagrams, these are known as *reciprocal figures.* The figure *pqrs* is known as the *funicular polygon*, and the point *o* as the *pole.*

It is worth noting, however, that the forces AB,

BC, CD and DA are fixed in magnitude and direction, and hence we can have only one force polygon, while the lengths of the links may vary, and consequently we may have any number of funicular polygons, and if we drew out a few different cases we would find that, for this particular system of forces, the figure *abcd* would remain the same, while the position of *o* would vary, both force and funicular polygons always closing.

A little thought will show that the converse construction will also hold good. Suppose, for example, we have four non-concurrent forces in equilibrium, and we wish to draw out the shape taken up by any cord under the action of these forces. We would begin by drawing out our force polygon, selecting a pole *o*, to which we would join the angular points of the polygon. We would then draw in the funicular polygon with sides parallel to the radiating lines in the force diagram, and we would find that the funicular polygon formed a closed figure, each corner of which fell on the line of action of one of the forces.

Resultant of Non-concurrent Forces.—This latter construction, as just described, is the one we make use of when finding the magnitude and direction of the resultant of a system of non-concurrent forces, and to make the application of this construction as clear as possible, we will now consider the following case.

Three forces, P, Q and S, are acting as shown in Fig. 29, and it is required to find their resultant in magnitude and direction.

Draw out first the force polygon *abcd*, and, since the forces are not in equilibrium, this polygon will not be a closed figure, and hence the closing line *ad* will represent the magnitude of the resultant R.

Select a pole *o*, and join *oa*, *ob*, *oc* and *od*, then from

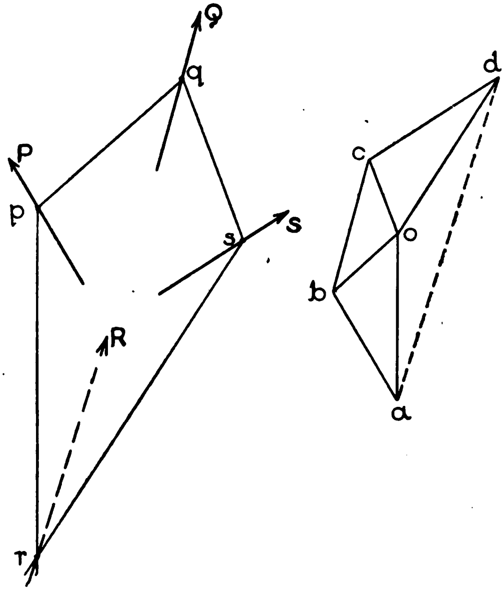


Fig. 29.

p, any point in the line of action of the force P, draw a line *pr* parallel to *oa*; from *p* draw *pq* parallel to *ob*, and so on, until the last line *sr* parallel to *od* cuts *pr* in the point *r*. Through *r* draw a line parallel to *ad*, and this line represents the line of action of the

resultant, the sense, as indicated by the arrowhead, being determined by following the forces round the polygon, although, in this case, such a proceeding is unnecessary, as the sense is obvious.

Resultant of Like Parallel Forces.—A particular case of the above is its application to the finding of the resultant of a system of like parallel forces.

Set down the forces AB, BC, CD, DE and EF. (It should be noted that the force polygon, in this

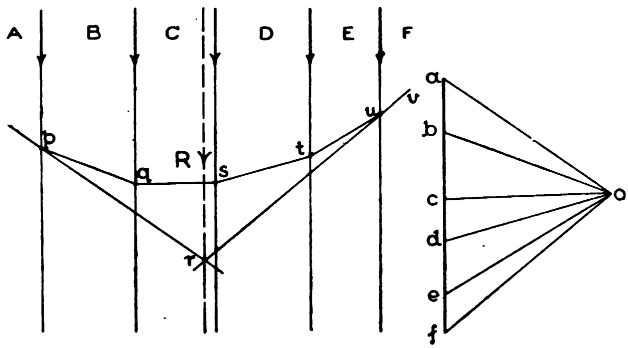


Fig. 30.

instance, is a straight line, and that the closing line *af* represents the magnitude of the resultant.)

Select a pole *o*, and join *oa*, *ob*, *oc*, *od*, *oe* and *of*, and then from *p*, any point on the line of action of AB, draw, in the space A, a line *np* parallel to *oa*; from *p* draw, in the space B, a line *pq*, parallel to *ob*, and so on, until *of* is reached in the line *uv*, in the space F.

Produce *np* and *vu* to intersect in *r*, and through *r* draw a line parallel to *af*.

Then this line R represents the line of action of the resultant, its magnitude being represented by *af* —the sum of the downward forces.

Resultant of Unlike Parallel Forces.—The case of unlike parallel forces will be easily understood from an examination of Fig. 31 without much further explanation. The only point calling for any particular care is the setting out of the forces in the force polygon.

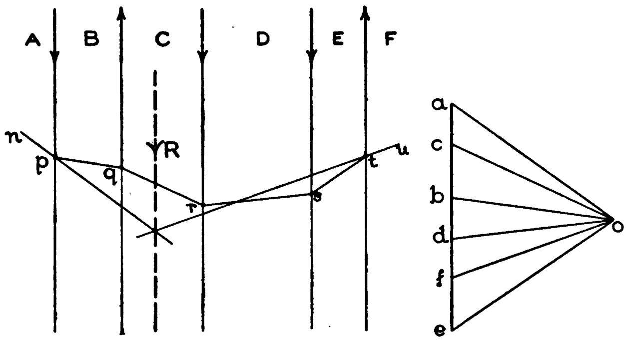


Fig. 31.

Thus, AB is set *down* as *ab*, but the next force BC is measured *upwards* as *bc*, the next two forces are measured *downwards* as *cd* and *de*, while the last force EF is measured *upwards* as *ef*.

Note also that the point *f* is below the point *a*, so that *af*, which represents the resultant, is measured *downwards*, and consequently acts as shown at R.

No difficulty should be encountered in drawing in the lines in their proper places if the rule adopted in

the previous question be made use of in this, viz., draw in the *space A* a line *parallel to oa,* and in the *space B* a line *parallel to ob*, and so on.

Moments.—If we examine Fig. 32, and consider the effect of P on the point O, we see that there is a tendency to cause rotation about the point O. This rotational effect depends on two things—the magnitude of P and the length of R—and is numerically equal to the product of P and R, measured in suitable units.

The tendency to rotation is known as *the moment of the force P about O,* and if P be measured in pounds and R in inches, then this moment equals PR lbs. in.

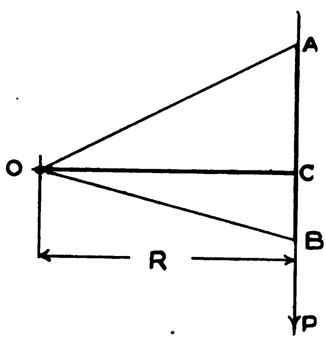


Fig. 32.

If we mark off, along the line of action of P, a length AB, equal to P, and join OA and OB, we get a triangle whose area is equal to $\frac{1}{2}AB \times CO$. The moment of P about O is equal to $P \times OC$, that is, it is equal to $AB \times OC$. Hence the moment of P about O may be represented by the area of the triangle AOB, if a suitable scale be devised.

Thus moments, which are the product of two quantities, may be represented by areas.

The force P would tend to produce a rotation about O in a *clockwise direction;* such is known as

a *negative moment.* A *positive moment* causes a *counterclockwise rotation.*

Graphic Representation of Moments. — The moment of a force can also be represented graphically by the aid of the funicular polygon. (Fig. 33.)

Let AB be the given force and O the point about which it is tending to cause rotation.

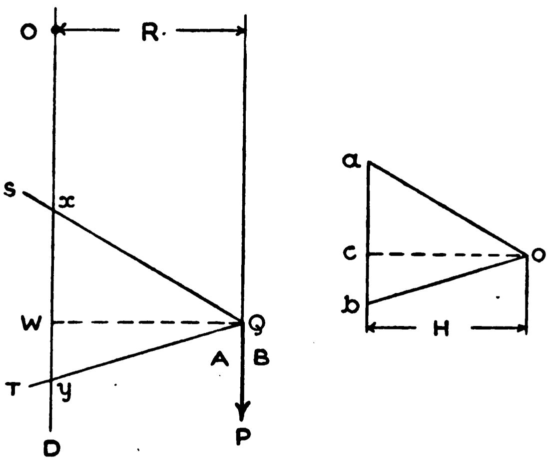


Fig. 33.

Draw *ab* parallel and equal to AB. Select a pole *o*, and join *oa* and *ob*, the position of *o* being so chosen that H is preferably an even number of inches. From Q, any point in the line of action of AB, draw two lines, QS and QT, parallel to *oa* and *ob* respectively, and through O draw a line OD parallel to AB, and cutting QS and QT in *x* and *y* respectively. Draw QW perpendicular to OD.

Comparing triangles Q*xy* and *oab*, we find they are similar, and hence

$$ab : xy :: oc : \mathrm{WQ}.$$
$$\therefore\ ab \times \mathrm{WQ} = xy \times oc.$$
$$= \mathrm{AB} \times \mathrm{WQ}.$$

Now $\mathrm{AB} \times \mathrm{WQ}$ equals the moment of P about O, and therefore $xy \times oc$ also equals the moment of P about O, and oc equals H, hence the moment of P about $\mathrm{O} = xy \times \mathrm{H} =$ intercept of first and last lines of the funicular polygon on the line through the point parallel to the force × polar distance.

The length xy must, of course, be measured to its proper scale. Thus, if 1 in. $= n$ lbs. (force scale) and 1 in. $= m$ ft. (space scale), then the moment of $\mathrm{P} = xy \times n \times m \times \mathrm{H}$ lbs. ft. about O. (See page 79.)

We know from our study of statics that the moment of the resultant of a system of forces is equal to the algebraic sum of the moments of the several forces about the same point, and that if a system of forces has no tendency to rotate about any point, then either the resultant is zero or, if not, then it passes through the given point.

Although the construction just given is of no great practical utility when dealing with a single force, yet the application of the underlying principles to the finding of the moment of the resultant of a system is one of extreme utility.

In the treatment of such problems we are, in general, reduced to two cases, in which we have to deal firstly with non-parallel forces, and secondly with parallel forces.

Moment of a System of Non-parallel Forces.—In Fig. 34 we are given a system of non-parallel forces, and we are required to find the moment of the system about O, or, what is the same thing, the moment of the resultant about O.

We have three distinct operations to perform in solving this problem. First, we must find the magni-

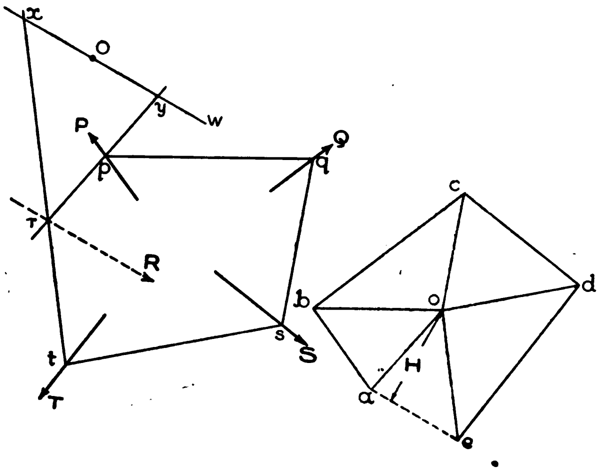


Fig. 34.

tude of the resultant; second, we must find its line of action; and third, we must find the moment of the resultant about O.

Draw the force polygon *abcde*, and join *ae*. The length *ae* gives the magnitude of the resultant. Select a pole *o*, and join *oa*, *ob*, *oc*, *od* and *oe*, the pole *o* being so chosen that H is an even number of inches. From *p*, draw *pr* parallel to *oa*, and *pq*

parallel to *ob*; from *q* draw *qs* parallel to *oc*, and so on, until the last line of the funicular polygon *tr* cuts the first line *pr* in the point *r*.

Through *r* draw a line R parallel to *ae*, and this line will represent the line of action of the resultant.

Through O draw a line OW parallel to R, and cutting *pr* produced in *y* and *tr* produced in *x*. Then the moment of the resultant equals $xy \times \mathrm{H}$ lbs. in. (xy being measured in pounds and H in inches). If the system of forces be drawn to a scale of 1 in. $= n$ lbs., then the moment of the resultant will equal xy in. $\times n \times \mathrm{H}$ lbs. in.

Moment of a System of Like Parallel Forces.—It will only be necessary to indicate the method of solution in the case of like parallel forces.

Produce *np* to cut OW in *x*, and produce *ut* in both directions to cut OW in *y* and *nx* in *r*. Through *r* draw a line parallel to *ae*. Then the moment of R about $\mathrm{O} = \mathrm{R} \times h = xy \times \mathrm{H}$.

From a study of Fig. 35 it will be seen that it is quite possible for the point O to be in such a position that the line through it parallel to *ae* would coincide with the line of action of the resultant R, and that if such be the case, then the resultant would have no moment about that point.

Couples.—A particular case of moments is the one in which we are dealing with two equal and unlike parallel forces. In Fig. 36 let the two forces be P and P and the perpendicular distance between them be R. Adopting the same treatment as in the previous problems, we begin

by drawing the force polygon *abc*. Draw first *ab* upwards to represent the upward force P, then *bc* downwards to represent the downward force P, and, since the two forces are parallel and equal, obviously the point *c* will coincide with *a*. Select a pole *o*, and join *ob* and *oa* (which is also *oc*). From *p* draw a

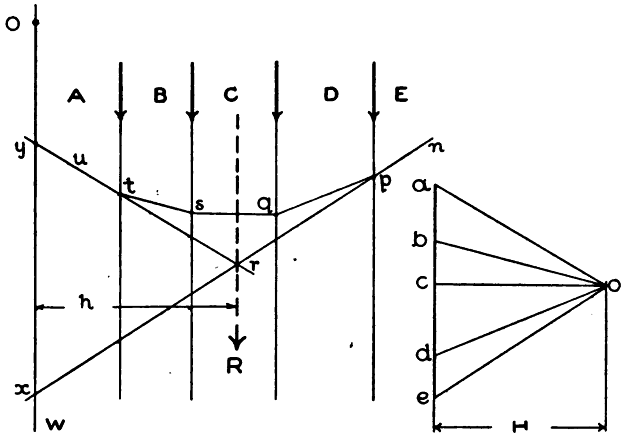


Fig. 35.

line *np* parallel to *oa*; from *p* draw a line *pq* parallel to *ob*, and finally a line *qs* parallel to *oc*.

It will now be noticed that, since the point *c* coincides with *a*, the force polygon is a closed figure, and hence one condition of equilibrium is satisfied, but we also notice that the lines *np* and *qs*, since they are drawn parallel to the same line, are parallel to one another, and would therefore never meet if produced. Hence it follows that our funicular polygon would

not close, and consequently the system is not in equilibrium.

If such a system be applied to a body, obviously motion of some kind will take place, the resulting motion being one of rotation without translation in any direction.

Such a system is known as a *couple*, the distance R being called the *arm of the couple.* Considering

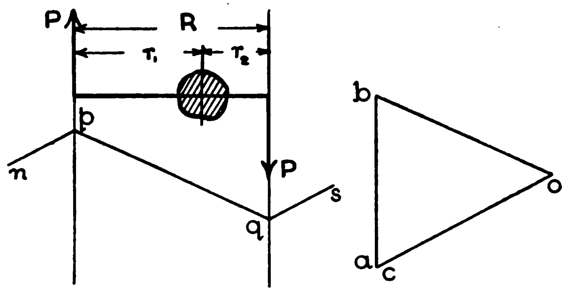


Fig. 36.

its effect on the given body, we see that the total moment is made up of two parts.

$$\text{Total moment} = (P \times r_1) + (P \times r_2).$$
$$= P(r_1 + r_2).$$
$$= P \times R.$$

$\therefore$ Total moment = the product of one force and the arm of the couple.

This product is known as the *moment of the couple*, while the direction of rotation is known as the *sense of the couple*, and is positive or negative according as the rotation is counterclockwise or clockwise.

The solution of problems dealing with couples will be readily obtained by the application of the methods adopted in the solutions in the preceding examples.

The moment of a couple is the same about all points in its plane.

A couple is measured by the magnitude of its rotational tendency, and two couples are equal if they both produce the same turning effect on a body, but if we have two couples, whose rotational tendencies are equal in magnitude but opposite in sense, the total effect produced is zero, or, in other words, the one couple neutralizes the other.

Hence, if we wish to balance or neutralize the effect of a couple, we must apply to the system an *equivalent couple*, that is a couple equal in moment but opposite in sense.

Examples.

1. Draw a regular pentagon of 1 in. side. Letter it clockwise ABCDE, beginning A at the apex point. Bisect the sides and raise perpendiculars to represent forces. The forces have the following magnitudes: On AB, 13 lbs.; on BC, 12 lbs.; on CD, 20 lbs.; on DE, 18 lbs.; and on EA, 14 lbs. Determine the magnitude of the resultant, its direction and the perpendicular distance of its line of action from the apex point.

2. Four forces, P, Q, S, T, act at the corners of a square ABCD. The magnitudes of the forces are $P=40$ lbs.; $Q=60$ lbs.; $S=30$ lbs.; $T=25$ lbs. The lines of action of the forces are such that the angle $P\hat{A}B=120°$; $A\hat{B}Q=150°$; $B\hat{C}S=135°$; $C\hat{D}T=120°$.

Determine the magnitude of the resultant, its inclination to AD and distance from A, at which it cuts AD produced if necessary.

3. Draw a square, lettered continuously PQRS, each side $1\frac{1}{2}$ in. long. Forces of 10, 8 and 4 lbs. act in the directions RP, SQ and QR respectively. Using a funicular polygon, determine the magnitude of the resultant of these three forces. State its magnitude in lbs., its perpendicular distance from P, and its inclination to PQ.

4. Draw a horizontal line and in it mark a point O. Mark off to the left distances OA $=\frac{1}{2}$ in. and OB $=1\frac{1}{4}$ in., and to the right distances OC $=1$ in. and OD $=2\frac{1}{4}$ in. Through A, B, O, C and D draw verticals to represent forces having the following magnitudes: $P=20$ lbs.; $Q=35$ lbs.; $S=28$ lbs.; $T=14$ lbs.; $W=40$ lbs., all forces acting downwards.

Determine the magnitude of the resultant and its position relative to the point O.

5. Three vertical forces, P, Q and S, act downwards on a body. The distance between P and Q is 4 ft., and between Q and S, 3 ft. The magnitudes of the forces are 80, 45 and 60 lbs. respectively.

Determine the position and magnitude of a single force which would relieve the body of the action of the three forces P, Q, S.

6. Considering the same system of forces as are given in Question 4, but assuming P, S and W to act upwards instead of downwards, determine now the magnitude and position of the resultant.

7. A lever, OA, 3 ft. long, is pivoted at O. It

carries at its end A, rigidly attached to it, a circular hoop 3 ft. diameter, the centre of the hoop coinciding with A. Forces P, Q, T, S act tangentially to the rim at the end of radii whose inclinations are 28°, 108°, 210° and 295° respectively. The magnitudes of the forces are 80, 93, 74 and 34 lbs. respectively.

Determine the moment of the system about O.

8. ABCD is a square of 3 in. side. Forces of 80, 90, 45 and 60 act outwards along the direction of the diagonals at the points A, B, C and D respectively.

Determine the moment of the system about the corner A of the square.

9. If in the previous question the forces had acted outwards along the directions of the sides (all tending to cause rotation of the square in the same direction), what would then have been the moment of the system about the centre of the square?

CHAPTER V

BENDING MOMENT AND SHEARING FORCE DIAGRAMS

In the foregoing pages we have, with a few exceptions in Chapter III., dealt only with the general principles involved in the solution of graphical problems, and, as the utility of graphics is most exemplified in its application to practical problems, we will now proceed to consider the application of these principles to the solution of simple problems involving the stresses in the members of various composite structures.

Before, however, passing on to the consideration of roofs and bridges, we might, with advantage, take up, at this point, the consideration of bending moment and shearing force diagrams for beams of various types and with various loadings.

Bending Moment.—*The bending moment at any section of a beam is equal to the algebraic sum of the moments of all forces acting either to right or left of the section.*

Possibly the best idea of what is taking place in a beam section, subjected to a bending moment, can be obtained from a study of the model shown diagrammatically in Fig. 37. Here we have a cantilever, from which a central section has been removed,

leaving the part ABCD detached from the part which is fixed into the wall. The cantilever carries a single concentrated load P at its outer extremity, and, by the addition of certain parts, we are able to keep the part ABCD in equilibrium and also in the same position which it occupied before the removal of the central section. We can now investigate the effects of these additional parts, and, in doing so, we will assume that thecantilever itself has no weight, and that we are dealing only with the load P and the effects produced thereby.

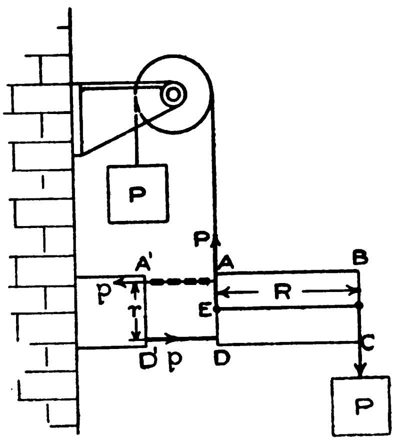


Fig. 37.

The first effect of P, which we notice, is that it tends to cause the portion ABCD to move bodily downwards, and, in order to counteract this tendency, we must apply an equal and opposite force. In the original beam, motion downwards would have been prevented by the resistance of the material on the section AD, and, in order to keep the conditions the same, we must apply our balancing force at this point. This is accomplished as shown, and it will be noticed that this force, whether artificial or real,

together with the load P, forms a couple whose moment is $P \times R$, and whose rotational tendency is clockwise.

This couple constitutes the bending moment on the section AD.

We have now this rotational tendency to deal with, and we have seen that this tendency can be neutralized only by a couple of equal and opposite tendency. Suppose, now, that ABCD starts to rotate about E, one of the effects produced is a lessening of the distance DD^1 and the other an increasing of the distance AA^1, and these two effects are neutralized in the model by introducing, at DD^1, a strut, and at AA^1 a tie in the nature of a chain. The force acting in the chain is a pull or tension, that in the strut a push or compression. These two forces acting at a distance r from one another form a couple whose moment is $p \times r$ equal to $P \times R$, but whose rotational tendency is counterclockwise. These two couples form a system in equilibrium. In the original beam, the push and pull, constituting the couple $p \times r$, are represented by the tensile and compressive stresses induced in the fibres of the material, while the upward force which we applied at AD is represented by the shearing resistance on AD. The effect of applying a force P is therefore twofold, for on any section AD we have induced a bending moment $P \times R$, which is neutralized by the resisting moment pr, and also a shearing force, which is neutralized by the shearing resistance.

Shearing Force.—*The shearing force on any section*

of a beam is equal to the algebraic sum of all forces acting either to right or left of the section.

Conventions in Use regarding B.M. and S.F.—We have just seen that, in the case above, the stress in the top layers of the cantilever was of the nature of a pull, and obviously, if the material be extensible, then those top layers would increase in length, while, for the opposite reason, the bottom layers would decrease in length, and the final shape of the cantilever would be as shown in Fig. 38 (*a*), the curve being *convex upwards;* in this case the bending moment is said to be *positive* and the diagram of bending moment is drawn *above the datum line.* Should the beam, however, take up the shape shown in Fig. 38 (*b*), that is, *concave upwards*, the bending moment is said to be *negative*, and the diagram is then drawn *below the datum line.*

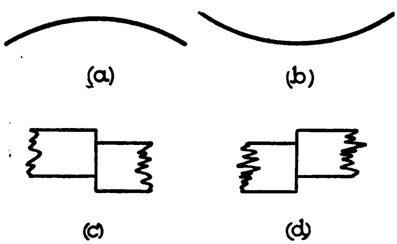


Fig. 38.

With regard to the shearing force, if any section of the beam tends to move upwards relative to an adjacent section to the right, as shown in Fig. 38 (*c*), the shearing force is said to be *positive*, and the diagram is drawn *above* the datum line, while, if the section tends to move down relative to its right-hand neighbour, as in Fig. 38 (*d*), the shearing force

is said to be *negative*, and the diagram is drawn *below* the datum line.

Construction of a Parabola.—In dealing with bending-moment diagrams the curve known as the parabola appears so often that it might be well, at this stage, to indicate two approximate methods of setting out the curve.

First Method.—Let AB, in Fig. 39, be the line on which the parabola is to be described. Bisect AB in E and draw EC perpendicular to AB and of such a length that EC represents the required depth of the parabola. Produce EC to D, making CD = EC. Join AD and BD and divide AB and DB into any number of equal parts, numbering them as shown, then join across the corresponding numbers. Trace in the required curve tangential to the cross lines.

Second Method.—As before, let AB be the given line, on which the parabola is to be described. Bisect AB in D and draw DC perpendicular to AB and equal to the required depth of the parabola. Complete the rectangle AEFB, divide EC and AE into the same number of equal parts, numbering them as indicated. Join the division points 1, 2, 3, 4 in the line AE to the point C. From the points 1, 2, 3, 4 in EC raise perpendiculars to cut the lines from the correspondingly numbered points in AE as shown Through the intersections so obtained trace the required curve.

Note that the accuracy of either method depends on the number of subdivisions: the greater the number the greater the accuracy.

Symmetrically Loaded Beams.—In many beam problems, where the loading is symmetrical or uni-

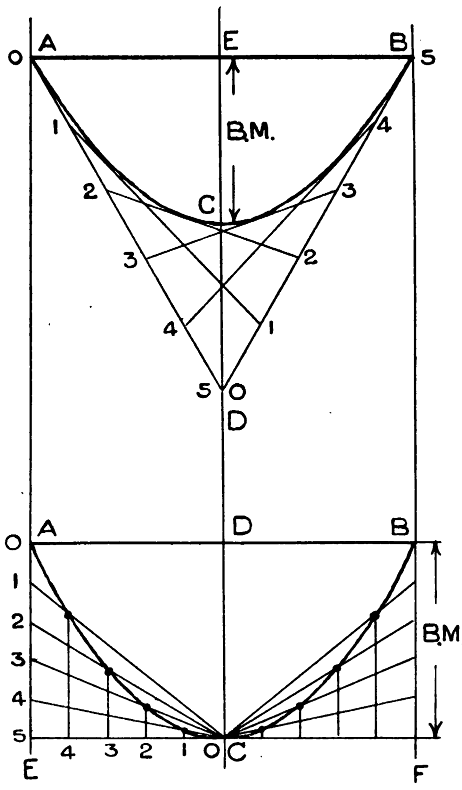


Fig. 39.

formly distributed, the drawing of the B.M. and S.F. diagrams presents little or no difficulty, and, when once the maximum bending moment has been deter-

mined, the diagram can be drawn without any special construction. We will deal with such problems first, and then proceed to investigate the method of applying the funicular polygon to the solution of the more difficult case of unsymmetrical loading.

Case I. Cantilever: Concentrated Load at Free End.—Considering a section of the beam at X, we see that the bending moment at that section is W $(L-x)$. This is obviously an expression of the first degree, and will therefore be represented by a straight line. The B.M. has its maximum value when $x=o$, or, in other words, the B.M. is greatest close to the wall. The B.M. is then equal to WL. Again when $x=L$, the B.M. becomes zero, and hence the B.M. at the free end equals zero.

Select a base or datum line AB, and set up, at A, a line AC to represent WL to some suitable scale; join CB. The triangle ABC is the bending-moment diagram, and it is drawn above the datum line AB, since the B.M. in this case is positive.

The shearing force at X is equal to W, and is the same for all sections, hence the S.F. diagram is a rectangle DEFG whose height DG represents W to scale. The S.F. is positive and is consequently drawn above the datum line DE.

Let us now suppose that we wish to find the B.M. and S.F. at any section, say X. From X we drop a perpendicular to cut the B.M. diagram in *ca* and the S.F. diagram in *gd*.

If the B.M. scale be such that 1 in.$=n$ lbs. ft., then $\text{B.M.}_X=(n \times ca)$ lbs. ft., and if the S.F. scale

be such that 1 in. $= m$ lbs., then the S.F.$_X$ = $(m \times gd)$ lbs.

Case II. Cantilever: Uniformly Distributed Load.—In this case the load is evenly distributed

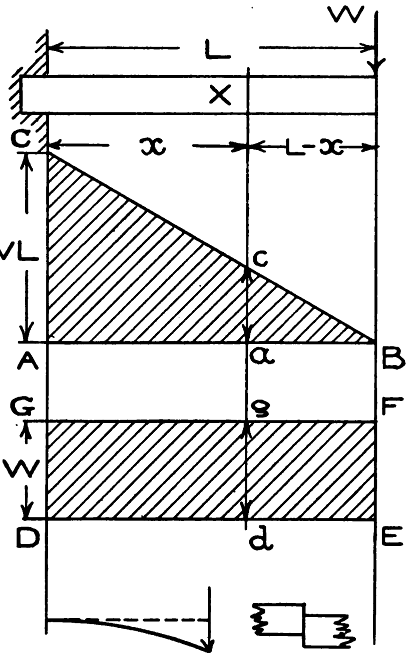


Fig. 40.

along the beam, and each foot length of the load is assumed to weigh w lbs. The total load W is therefore equal to $w \times$ L lbs. Considering, as before, the B.M. at a section X, and dealing with the load to the right of the section only, we see that the total load

acting to the right of the section equals $w\,(L-x)$. This load is uniformly distributed over a length

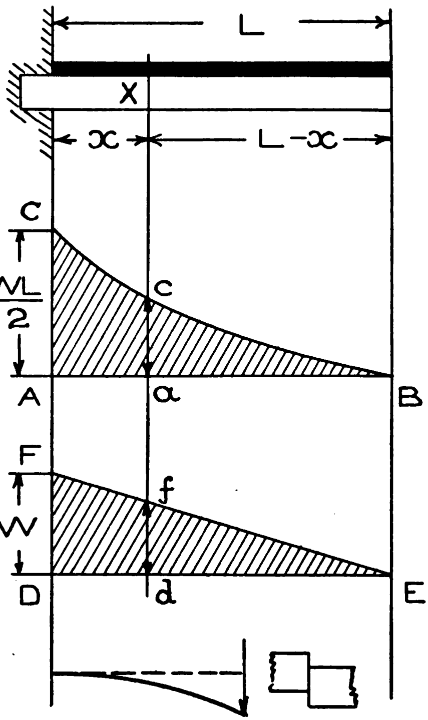


Fig. 41.

$(L-x)$ ft., and, since it acts through the centre of gravity it will act at a distance $\frac{L-x}{2}$ ft. from the section X.

$$\therefore \text{ B.M.}_X = w(\text{L} - x) \times \frac{\text{L} - x}{2} = \frac{w}{2}(\text{L} - x)^2.$$

This expression contains a squared quantity and is therefore of the second degree, hence the curve will be represented by a parabola.

The above expression is a maximum when $x = o$, and is then equal to $\frac{w}{2} \times \text{L}^2 = \frac{\text{WL}}{2}$.

Select a base line AB and set up AC equal to $\frac{\text{WL}}{2}$ to scale, and then, by method two of Fig. 39, describe the parabolic curve, having its apex at B.

The shearing force at X equals w $(\text{L} - x)$. This expression is of the first degree, and is a maximum when $x = o$, therefore $\text{S.F.}_{max} = w \times \text{L}$. It has its minimum value when $x = \text{L}$, and is then equal to zero. Our S.F. diagram is therefore a triangle FDE, as shown, in which the length DF represents wL or W to scale.

As before, *ca* and *fd* represent the B.M. and S.F. respectively at X, measured to their respective scales.

Case III. Beam simply supported at the Ends, and Loaded with a Concentrated Central Load.—In this case, as shown in Fig. 42, the load is central; so that each support will take an equal share of the load.

$$\therefore \text{ R}_1 = \text{R}_2 = \frac{\text{W}}{2}$$

$$\begin{aligned} \text{B.M.}_X &= \text{R}_2 \times \left(\frac{\text{L}}{2} - x\right) \\ &= \frac{\text{W}}{2} \times \left(\frac{\text{L}}{2} - x\right) \\ &= \frac{\text{W}}{4}(\text{L} - 2x) \end{aligned}$$

This expression is represented by a straight line, the maximum value occurring when $x=o$, *i.e.*, at the centre, while the minimum value occurs at the ends when $x=\frac{L}{2}$, the value then being zero. The maximum value $= \frac{WL}{4}$. The B.M. diagram, in this case, is a triangle, having its apex at the centre of the span; it is drawn *below* the datum line, since the B.M. in this instance is *negative*. The maximum depth of the diagram equals $\frac{WL}{4}$ to some scale.

The S.F. at any section in the left-hand half is obviously R_1 or $\frac{W}{2}$, and is constant in this half of the beam. Hence our S.F. diagram, for this half, is a rectangle of height $\frac{W}{2}$ (to scale), and is drawn *above* the datum line, since the S.F. in this half is *positive*.

Passing on from left to right, we find that, as soon as we pass over the centre, the value of the S.F. alters, for now we have two loads to deal with, and, in estimating our S.F., we must take their algebraic sum. We have $\frac{W}{2}$ acting upwards and W acting downwards, so that the S.F. is equal to $+\frac{W}{2} - W = -\frac{W}{2}$.

This value is constant from the centre to the right-hand support, and is therefore represented by a rectangle of depth $\frac{W}{2}$, but this time drawn *below* the datum line.

It will be noticed, therefore, that the S.F. passes from a positive to a negative value, and, in so doing, passes through a zero value, and it is worth noting

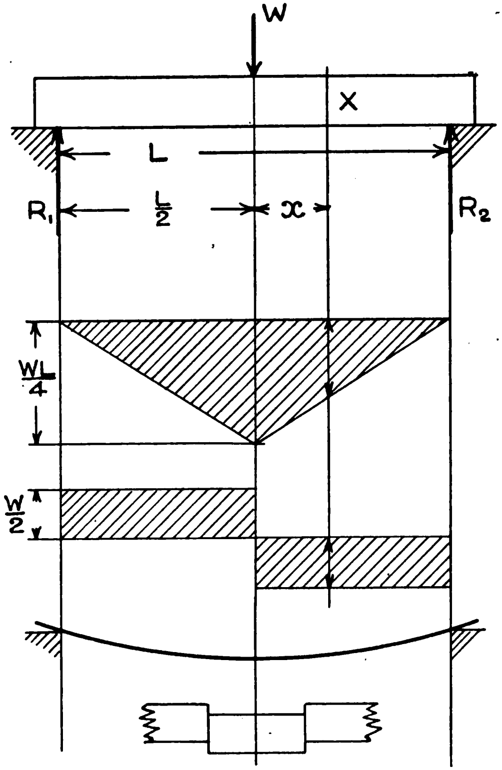


Fig. 42.

that the point of zero S.F. coincides with the point of maximum B.M.

Case IV. Beam simply supported at the Ends

Carrying a Uniformly Distributed Load.—The load in this case, as shown in Fig. 43, is uniformly distributed across the span, and, as in the previous case, the reactions, R_1 and R_2, are each equal to half the total load. The load per foot-run may be taken as w lb., hence the total load is equal to $w \times L$ lbs. Each reaction is therefore equal to $\frac{wL}{2}$, or $\frac{W}{2}$, where $W = w \times L$.

Considering the B.M. at any section X distant x from the centre-line, we find that we have two loads acting to the right of the section, viz., an upward load $R_2 = \frac{wL}{2}$ acting counterclockwise at distance $\left(\frac{L}{2} - x\right)$ from X, and a downward load consisting of a piece of the total load of length $\left(\frac{L}{2} - x\right)$, whose weight is $w\left(\frac{L}{2} - x\right)$, acting clockwise at a distance $\frac{1}{2}\left(\frac{L}{2} - x\right)$ from X.

$$
\begin{aligned}
\therefore \text{B.M.}_x &= \left\{R_2 \times \left(\frac{L}{2} - x\right)\right\} - \left\{w\left(\frac{L}{2} - x\right) \times \frac{1}{2}\left(\frac{L}{2} - x\right)\right\} \\
&= \left\{\frac{wL}{2} \times \left(\frac{L}{2} - x\right)\right\} - \left\{\frac{w}{2}\left(\frac{L}{2} - x\right)\left(\frac{L}{2} - x\right)\right\} \\
&= \left\{\frac{w}{4}\left(L^2 - 2xL\right)\right\} - \left\{\frac{w}{8}\left(L - 2x\right)\left(L - 2x\right)\right\} \\
&= \frac{w}{8}\left(2L^2 - 4xL - L^2 + 4xL - 4x^2\right) \\
&= \frac{w}{8}\left(L^2 - 4x^2\right) \\
&= \frac{w}{2}\left(\frac{L^2}{4} - x^2\right)
\end{aligned}
$$

It appears from this expression that the B.M. diagram is a parabola. The expression becomes zero when $x = \pm\frac{L}{2}$, hence at the supports the B.M. is zero. The expression has its maximum value when $x=o$, *i.e.*, at the centre, the value then being $\frac{wL^2}{8}$ or $\frac{WL}{8}$. The drawing of the B.M. diagram should now present no difficulty.

The S.F. at X is equal to the algebraic sum of the two forces acting to the left (or right) of the section.

$$\text{S.F.}_X = \frac{wL}{2} - w\left(\frac{L}{2} - x\right)$$

$$= wx.$$

The S.F. is therefore represented by a straight line, and when $x=o$, the S.F.$=o$, this value occurring at the centre. The maximum values occur when $x = \pm\frac{L}{2}$, the values then being $\frac{wL}{2}$, and occurring at the supports. The construction of the diagram will be readily understood from the adjoining figure. The solutions of the four typical cases which we have been considering hardly merit the term "graphical solutions," since the *calculation* of the maximum and minimum values of the B.M. and S.F. was performed before setting out the diagrams, still the constructions adopted are quite justifiable in view of the symmetry of the loadings. Examples so nicely arranged are the exception rather than the rule in practice, the arrangement and loadings in practical cases being more or less unsymmetrical.

We will now proceed to investigate one or two cases of unsymmetrical loading, making use of the funicular polygon in constructing the B.M. diagram.

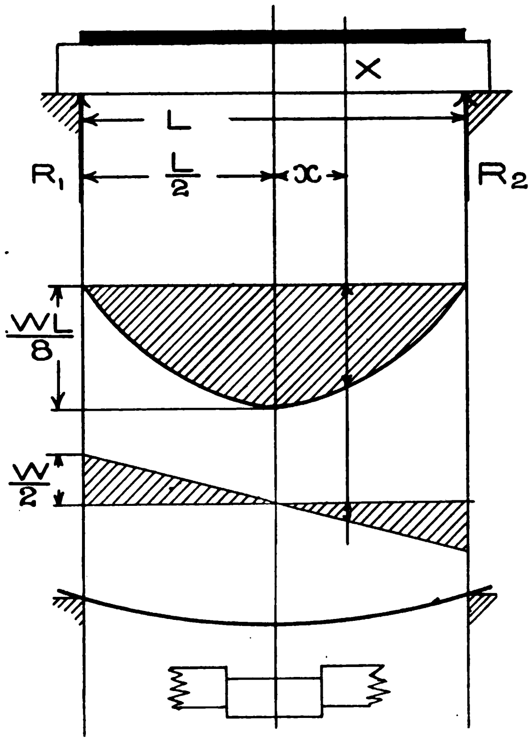


Fig. 43.

Unsymmetrically Loaded Beams.—Let us consider first the case of a simple cantilever loaded with a series of concentrated loads AB, BC, CD, and DE.

Set down to scale the loads *ab*, *bc*, *cd*, *de*, and from *e* draw *eo* perpendicular to *ae*, making its length an even number of inches. Join *ao*, *bo*, *co* and *do*. In the position diagram produce the lines of action of the loads as shown. In the space A draw any line *pq* parallel to *ao*; in the space B a line *qr* parallel to *bo*, and so on, closing the polygon with the horizontal line *tu* parallel to *eo*. The figure *pqrstu* now represents the B.M. diagram.

The shearing force diagram is constructed by projecting along the loads as shown from the load-line *ae*.

Note.—It is not necessary that *eo* be drawn perpendicular to the load-line, but it gives a neater B.M. diagram.

Scales of B.M. and S.F.—The diagrams, so constructed, are more or less worthless unless we know the scale by which the ordinates must be measured in order to determine the B.M. and S.F. at any section. The scale of B.M. is derived as follows: Suppose the scale to which the length of the cantilever has been drawn is 1 in. $= m$ ft., and that the load-scale to which *ae* has been drawn is 1 in. $= n$ lbs., and further, that the polar distance is H in. (*N.B.*—By the polar distance we mean the perpendicular distance of the point *o* from *ae*), then in our B.M. scale

$$1 \text{ in.} = m \times n \times \text{H lbs. ft.}$$

For example, suppose L to have been drawn to a scale 1 in. = 10 ft., and that the load-scale is 1 in. = 50 lbs., and H = 4 in., then

$$1 \text{ in. (B.M. scale)} = 10 \times 50 \times 4 = 2000 \text{ lbs. ft.}$$

To find the B.M. at X we drop a perpendicular and measure the intercept between *rs* and *tu*; then, say this measures 1·3 in., the B.M.$_x$ is consequently equal to $1·3 \times 2000 = 2600$ lbs. ft.

To show that this diagram really represents the B.M. diagram, we might consider it a little more fully,

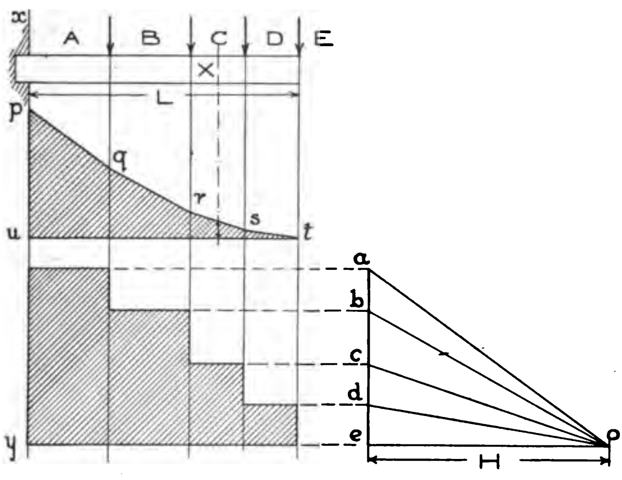


Fig. 44.

especially as the principle underlying this case is applicable in all others. We saw, in Chapter IV., that the moment of a system of forces about a point was equal to the intercept of the first and last lines of the funicular polygon on a line through the point parallel to the resultant force, multiplied by the polar distance.

Considering now the section close to the wall, we

have, at that section, a line xy parallel to the resultant of the loads. The first line of the funicular polygon is pq, and the last is tu, hence the intercept is the line pu. The moment of the system about this section is therefore $pu \times \mathrm{H}$, pu representing a force and H a distance.

It will be obvious that if L had been doubled, other things remaining as before, then the length pu would also have been doubled, so that if L be reduced to some definite scale, then the true length of pu will have been reduced in the same ratio. Hence, if 1 in. $= m$ ft., it follows that our diagram is reduced $12m$ times, and, therefore, the full-size length of pu would be $(pu \times 12m)$ ins. Again, each inch represents n lbs., and consequently the magnitude of the force represented by pu is $(pu \times 12m \times n)$ lbs. The moment of this force about the section is therefore $pu \times 12m \times n \times \mathrm{H}$ lbs. ins., or $pu \times 12m \times n \times \frac{\mathrm{H}}{12}$ lbs. ft., which is equal to $pu \times m \times n \times \mathrm{H}$ lbs. ft.

This may be written as—

The depth of the B.M. diagram under the section $\times$ the B.M. scale.

Note carefully that, although the result is expressed as lbs. ft., the distance H is measured in inches and not in feet. Confusion often arises over this point, but a careful study of the above reasoning will overcome any difficulty which may arise.

Simply supported Beam with Concentrated Loads.—Let us consider now the case as shown in Fig. 45, in which we have a simply supported beam

carrying concentrated loads AB, BC and CD. Produce the lines of action of the forces and set down the lines *ab*, *bc* and *cd* to represent the loads. Select a pole *o*, so that H is an even number of inches. (In the majority of cases the best position for *o* is that which makes *aod* approximately an equilateral triangle.) Join *ao*, *bo*, *co* and *do*. From *p* any point in the line of action of R_1 draw, in the space A, a line *pq* parallel to *oa*; from *q* draw, in the space B, a line *qr* parallel to *ob*, and so on until the point *t* is reached. Join *pt*, then the closed figure *pqrst* is the bending-moment diagram for the above beam.

From *o* draw *oe* parallel to *pt*, and the lines *de* and *ea* represent the reactions R_2 and R_1 respectively.

The shearing force diagram is constructed by projecting across from the load-line as indicated. Considering the S.F. in the left-hand half of the beam, we find that the S.F. from R_1 to AB is constant and equal to R_1, being also positive, so that it can be represented by a rectangle whose height is R_1 or *ea* on the load-line. Passing now to the right, the S.F. between AB and BC is represented by the sum of R_1 acting upwards and AB acting downwards, or on the load-line by a force *ea* acting upwards and *ab* acting downwards. The nett S.F. between AB and BC is therefore *eb* acting upwards, and hence our S.F. diagram for this part is a rectangle of height *eb*. The remainder of the diagram requires no further explanation.

Overhung Beam with Concentrated Loads.—The following example is well worth careful con-

sideration, embodying, as it does, a combination of the two previous cases. Set down the loads *ab*, *bc*, *cd* and *de*, select a pole *o*, and join up the points. In the space A draw a line parallel to *oa*. Difficulty usually arises at this point as to which space is the space A.

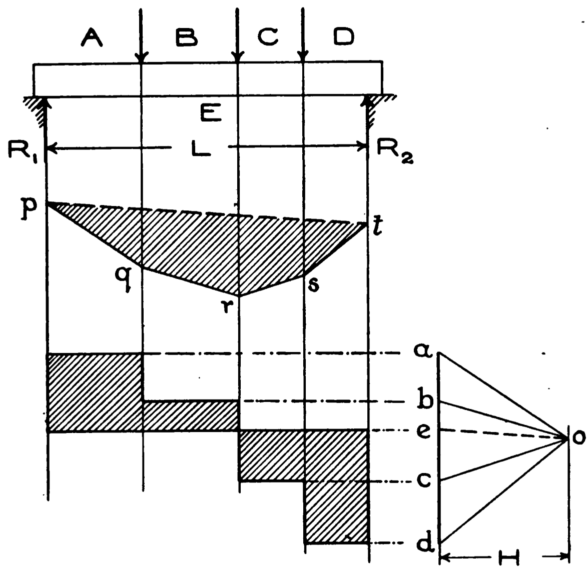


Fig. 45.

If, however, we follow round the force polygon, we have *ab*, *bc*, *cd*, *de*, and finally to close the polygon we have *ea*. Now we notice that the line *oa* is drawn to the point, *a*, which is the meeting-point of the lines *ea* and *ab*, or of the lines *fa* and *ab*. These latter lines represent R_1 and AB, and the line parallel to *oa* must

connect R_1 and AB in the position diagram. The line *pq* parallel to *oa* is the line required. The drawing of the other lines should now present no difficulty.

It will be noticed in this case that we have a crossed diagram. The points *x*, *x*, denoting zero B.M., are called *points of inflection*, and indicate a

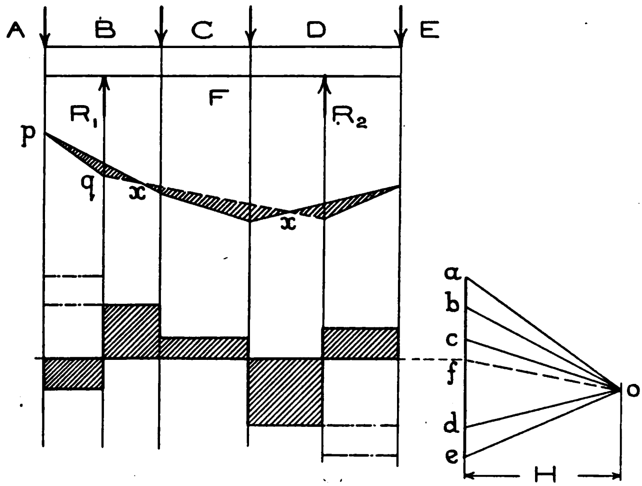


Fig. 46.

change of curvature in the beam at these points. In constructing the S.F. diagram it is worth noting that the points of inflection are coincident with sections in which the S.F. is a maximum.

Simply supported Beam: Compound Loading.—The following case, as shown in Fig. 47, indicates the method to be adopted when constructing the B.M. diagram for a beam with compound loading.

Draw out first the B.M. diagram for the concentrated loads AB, BC and CD as shown by the link polygon *pqrst*. Then calculate the maximum bending moment at the centre of the span, due to the uniformly distributed loading, and construct the B.M. diagram as shown by *uwv*, care, however, being taken to construct both diagrams to the same scale. We must now combine these two diagrams to form a single diagram of B.M. Draw a horizontal line *xy* and divide the span of the beam into a number of equal parts, and raise ordinates as shown. Considering now the first ordinate from the left-hand support, this ordinate passes through a section X of the beam. Now, at X, we have a B.M., due to the concentrated loads, equivalent to *ef*, and a B.M., due to the distributed load, equal to *gh*. The total B.M. at this point then is equal to $(ef+gh)$. From the point *k*, where this ordinate cuts the line *xy*, we mark off a distance *kl*, equal to $(ef+gh)$, and the point *l* gives us one point on our new B.M. diagram. All the ordinates are treated in a similar manner and the curve finally drawn through the points obtained.

The shearing force diagrams for each system of loading are first drawn out and the ordinates are then added to form the combined S.F. diagram.

The example as shown in Fig. 48 illustrates another case of compound loading. The constructions employed should be apparent from the diagrams without any detailed explanation. A symmetrically loaded beam has been taken in this case, but any system of loading with a beam of this type can

quite readily be treated in stages by the methods employed in the preceding examples.

It should be noted that a uniformly distributed load on a beam can be treated by the funicular polygon method. The load on the beam is divided

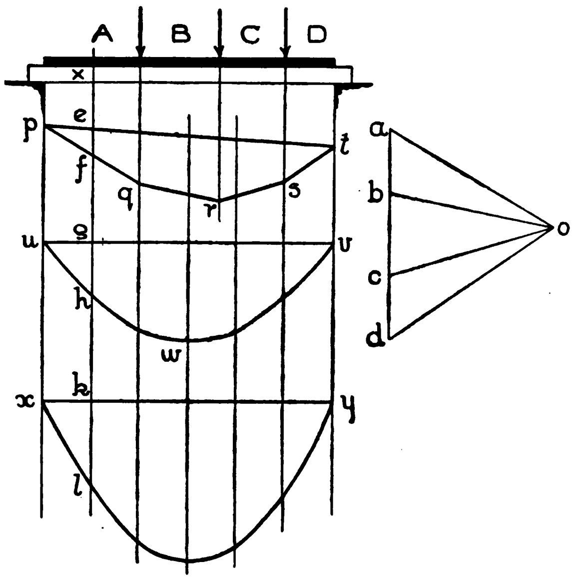


Fig. 47.

into a large number of equal parts, and these small parts of the load are treated as concentrated loads acting at their centres of gravity. The student should work out this case for himself, a proceeding which should be readily accomplished without any further explanation.

Examples.

1. Draw a horizontal line AB, 4 ins. long, and on it describe a parabola having its vertex $2\frac{1}{4}$ ins. above AB.

2. A cantilever is 14 ft. long and carries at its free end a load of 2 tons. Draw the B.M. and S.F. diagrams, and determine by measurement the B.M. at points 2 ft., 5 ft. and 8 ft. from the free end. Check by calculations.

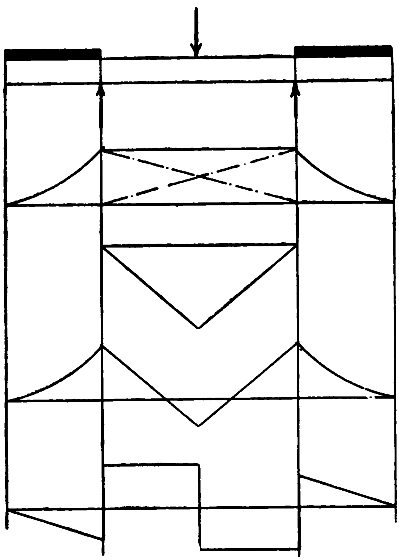


Fig. 48.

3. If the cantilever in the above question had carried a uniformly distributed load of $\frac{1}{2}$ ton per foot, what would have been the maximum B.M.? Draw the B.M. and S.F. curves and determine by measurement the B.M. and S.F. at 3 ft. and 9 ft. from the free end, checking your results by calculation.

4. A beam, 30 ft. span, is simply supported on two abutments. It carries a load of 3 tons at the centre of the span. Draw the B.M. and S.F. diagrams, and

determine the value of the B.M. and S.F. 6 ft. from the left-hand support.

5. Assuming the above beam to carry a uniformly distributed load of 600 lbs. per foot-run, what would be the maximum B.M.? Draw the curves of B.M. and S.F. and determine the values at a section 8 ft. from the right-hand support.

6. A beam, 20 ft. long, projects from a wall. This beam carries a platform which rests on battens placed at intervals of 4 ft., the first being placed on the free end of the beam. Starting from the free end, the loads coming on the battens are 400, 600, 700, 900, and 500 lbs. respectively. Draw the curves of B.M. and S.F. and determine the B.M. and S.F. 6 ft. and 10 ft. from the free end. What is the maximum B.M.?

7. A beam, 40 ft. span, rests on two abutments. Cross joists are fixed to this beam at points 4, 17, 24, and 32 ft. from the left-hand support, the corresponding loads being 2, 3, 2·5 and 4 tons respectively. Draw the diagrams of B.M. and S.F., and determine the values of the B.M. and S.F. at the centre of the beam, also at points 10 and 26 ft. from the left-hand abutment.

8. A beam, 40 ft. long, rests on two columns 28 ft. apart. On the left-hand end the overhang is 7 ft., and on the right-hand end 5 ft. A load of 4 tons rests on the left-hand overhang, and 3 tons on the right hand, both at the extreme ends. Two loads of 2 tons each are placed on the centre span, one 8 ft. from the left-hand support and the other 10 ft. from the right hand. Draw the B.M. and S.F. diagrams and determine the B.M. and S.F. at the centre of length of the beam.

9. A beam, weighing 40 lbs. per foot, covers a span of 36 ft. It carries a uniformly distributed load of 200 lbs. per foot-run, in addition to 3 loads, each weighing 1 ton, placed at 8, 14 and 28 ft. from the left-hand support. Draw the B.M. and S.F. diagrams. What is the maximum B.M., and where does it occur? What is the B.M. and S.F. 10 ft. from the right-hand support?

10. Assuming the beam in Question 8 to carry a uniformly distributed load of 150 lbs. per foot-run, in addition to the dead loads given, determine graphically the B.M. and S.F. at the centre of length of the beam. What is the maximum B.M. and S.F., and where does it occur?

CHAPTER VI

BEAMS WITH ROLLING LOADS

In all the cases which we have considered so far, we have seen that the loads were either concentrated at fixed points or uniformly distributed along the length of the beam. Such loads are known as *dead loads*. In many cases in practice, however, more especially in bridges, crane girders, and such structures, the loads, instead of being fixed in position, are free to move along the beam. Such loads are known as *rolling, or live loads*.

Single Concentrated Rolling Load.—We will now examine a few cases of rolling loads, considering first the case of a single concentrated load rolling along the simply supported beam, assuming the load to roll from R_1 to R_2, as shown in Fig. 49. Considering the B.M. at a section X distant x from the left-hand support, we have—

$$R_1 \times L = W(L - x)$$
$$\therefore \ R_1 = \frac{W(L-x)}{L}$$

The maximum B.M., with a concentrated dead load, always occurs just under the load, so that for the

given position of the load the maximum B.M. occurs at X, and hence we have—

$$\begin{aligned} \text{B.M.}_X &= R_1 \times x \\ &= \frac{W(L-x)}{L} \times x \\ &= \frac{W(Lx-x^2)}{L} \end{aligned}$$

This expression gives the B.M. for any section as

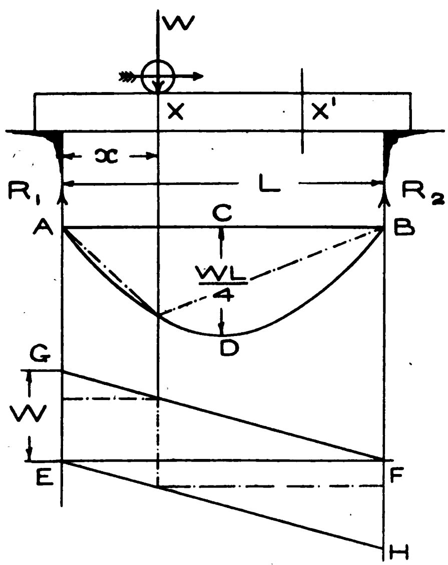


Fig. 49.

the load rolls across, and, since it is of the second degree, the curve, which shows the variation in B.M.

from one side to the other, must be a parabola. The above expression has its minimum values when $x=o$ and L, being then, in each case, equal to zero. It has its maximum value when $x=\frac{L}{2}$ and

$$\text{B.M.}_{\text{Centre}} = \frac{W\left(\frac{L^2}{2}-\frac{L^2}{4}\right)}{L}$$

$$=\frac{WL}{4}$$

This is the value of the maximum B.M. which we obtained for a concentrated dead load on a simply supported beam, as treated in the previous chapter. Draw a horizontal AB, and set down, at the centre of the span, a perpendicular CD, equal to $\frac{WL}{4}$, and construct a parabola, having D as its vertex. The B.M. diagram for any section is a triangle, having its apex below the load, as shown dotted, and if we draw out the B.M. diagrams for successive positions of the load, we will find that the apex of the triangle will always fall on the curve ADB. The curve ADB envelops all the B.M. diagrams, and is, on that account, often spoken of as the *envelope curve of B.M.*

The shearing force just to the left of X is obviously equal to $+R_1$ or $+\frac{W(L-x)}{L}$. This, being an expression of the first degree, shows that the envelope curve is a straight line. The shearing force is obviously a maximum when $x=o$, and is then equal to $+W$;

when $x = L$ the S.F. is equal to zero. Draw a horizontal line EF, and at E set up a line EG, equal to W, and join GF. The line GF is the *envelope of the positive shearing force.* In the same way it can be shown that the shearing force, just to the right of the load, is equal to $\frac{W(L-x)}{L} - W = -W\frac{x}{L}$. This expression has its maximum value when $x = L$, and is then equal to $-W$, and when $x = o$ its value is then zero. We now set down from F a line FH, equal to $-W$, and join EH. The line EH is the *envelopė of the negative shearing force.*

As the load rolls from R_1 to R_2 the variation in the positive shearing force (or the shearing force to the left of the load) is shown by the triangle EGF, while the variation in the negative shearing force (or the shearing force to the right of the load) is shown by the triangle EFH. It will be obvious that in measuring the shearing force at any section we must take the maximum S.F., whether it be positive or n egative.

It should be noted that, if we consider any section, say X_1, the shearing force at that section will be positive if W is approaching from R_2, while, if W approaches from R_1, the shearing force will then be negative.

Uniformly Distributed Rolling Load.—The next case which we shall consider is that in which we have a uniformly distributed load, weighing w lbs. per foot-run, and of length L moving across a simply

supported beam of span L. Examining Fig. 50, we see that—

$$\text{B.M.}_x = R_2(L - x)$$

$$\text{and } R_2 \times L = wx \times \frac{x}{2}$$

$$\therefore R_2 = \frac{wx^2}{2L}$$

$$\therefore \text{B.M.}_x = \frac{wx^2}{2L}(L - x).$$

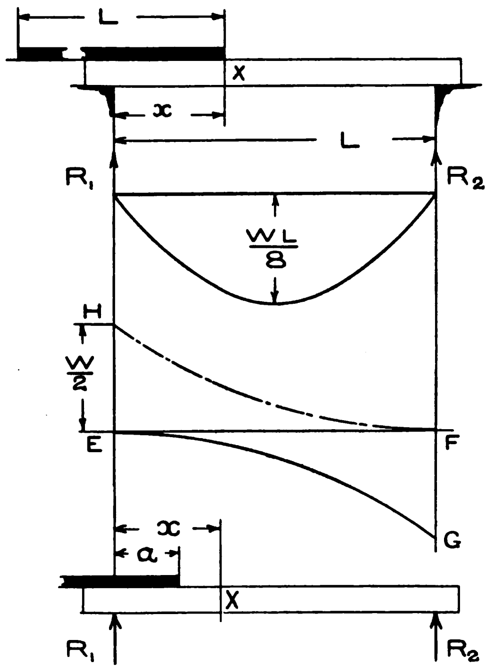


Fig. 50.

This expression, of the second degree, has its mini-

mum values when $x=o$ and L, being then, in each case, equal to zero. The magnitude of the B.M. gradually increases as the load moves from left to right. The maximum B.M. occurs at the centre of the span, when the span is fully loaded, and is then equal to $\frac{WL}{8}$; the parabolic curve, as shown, being the envelope of maximum B.M. This is the value of the maximum B.M. which we obtained for a uniformly distributed load on a simply supported beam, and hence the envelope curve of B.M. is exactly similar to the B.M. curve for a simply supported beam with a uniformly distributed load. When we come to consider the S.F., we will assume that the load has only travelled on to the beam a distance a (*see lower figure*). The S.F. at X is obviously equal to R_2.

$$\text{Now} \quad R_2 \times L = wa \times \frac{a}{2} = \frac{wa^2}{2}$$

$$\therefore \quad R_2 = \frac{wa^2}{2L}$$

The value of R_2 obviously depends on the length of a and increases as a increases. It will have its greatest value when a becomes equal to x, or, in other words, the S.F. at X will be a maximum when the load fills the beam right up to the section. The maximum $\text{S.F.}_x = \frac{wx^2}{2L}$, and is negative. The expression shows the envelope curve of negative S.F. to be

a parabola, and, when $x=$ zero, the S.F. is zero, while, when $x=\mathrm{L}$, the S.F. is equal to $\frac{w\mathrm{L}}{2}$, or $\frac{\mathrm{W}}{2}$. It will be noticed that as the load travels from left to right the S.F. just in front of the load is negative. In setting out the envelope curve, we first draw a base line EF, then set down FG to represent $\frac{\mathrm{W}}{2}$, and construct the parabolic curve as already explained.

If, on the other hand, the load had been travelling from right to left, a little calculation will show that, in this case, the S.F. just in front of the load is positive. The envelope curve of positive S.F. is shown in the figure by EHF.

In many cases in practice we have, in addition to a rolling load which may pass along the beam from either side, certain dead loads fixed at definite points on the span. Let us now consider the case, as shown in Fig. 51, in which we have two dead loads, AB and BC, acting on the beam in addition to a rolling load —of greater length than the beam—which moves from left to right. We first draw, by means of the funicular polygon, the B.M. diagram for the dead loads, as shown in *pqrs*. The envelope curve of B.M. is next drawn for the rolling load, this being the parabola *uvw*. These two diagrams are now combined, by adding corresponding ordinates, to give the combined B.M. diagram. From this latter curve we can now determine the maximum B.M. and the section at which it occurs.

It should be carefully noted that both of the B.M.

diagrams, *pqrs* and *uvw*, must be drawn to the same B.M. scale. In order to obviate the necessity of calculating the maximum bending moment in the case

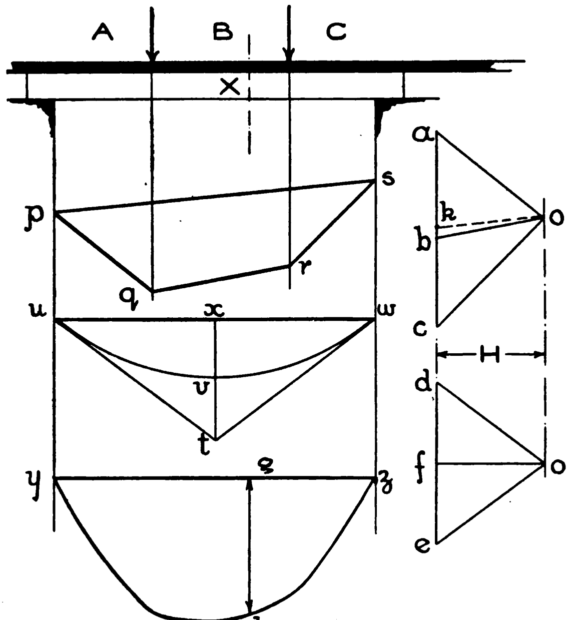


Fig. 51.

of the rolling load, the graphical construction might, with advantage, be applied in this case.

Set down a line *de* to represent $w \times L$, the maximum value of the rolling load which can come on the span. Draw *fo* the perpendicular bisector of *de*, making *fo* equal to H. Join *do* and *eo*. From *u* draw *ut* parallel to *do*, and from *w* draw *wt* parallel to *oe*.

Draw *tx* perpendicular to *uw* and bisect it in *v*. Using *ut* and *wt* as construction lines, construct the parabola *uvw*, having its vertex at *v*.

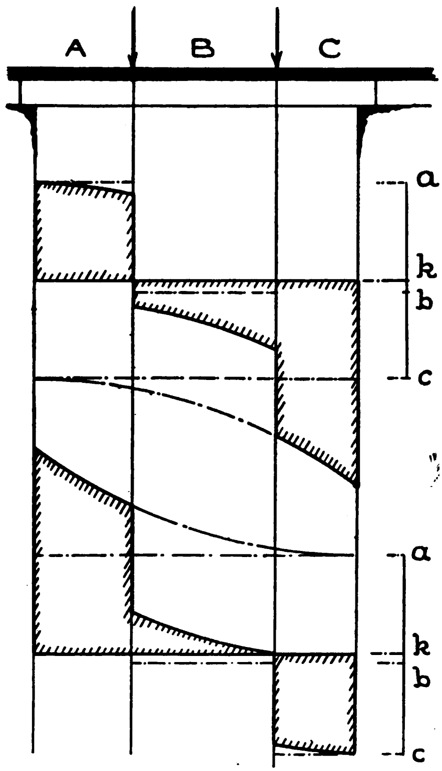


Fig. 52.

Draw a new base line *yz*, and on this, by the addition of ordinates, as previously explained, construct the combined diagram for the dead and rolling loads.

The greatest B.M. which can occur at any section, say X, is equal to *gh*, multiplied by the B.M. scale. $B.M._X = gh \times m \times n \times H$ lbs. feet.

The shape of the shearing force diagram will depend on the direction of motion of the rolling load. We first draw out the S.F. diagram for the dead loads, then construct the diagram for the rolling load, assuming it to move across the span first in the one direction and then in the other.

By adding corresponding ordinates we can construct the combined S.F. diagram. Fig. 52 shows the required diagrams, the upper diagram representing the case when the load moves from left to right, and the lower diagram when it moves from right to left.

CHAPTER VII

ROOF TRUSSES—DEAD LOADS ONLY

Framed Structures.—In dealing with the more practical application of Graphics to the determination of the stresses induced in the various members of roof trusses and bridge girders by the application of external loads, we have to consider what are known as *framed structures.*

A Frame is a structure consisting of several bars jointed together at their ends by pins, which allow of free motion in one plane round their centres. The several bars composing a frame are known as *members.*

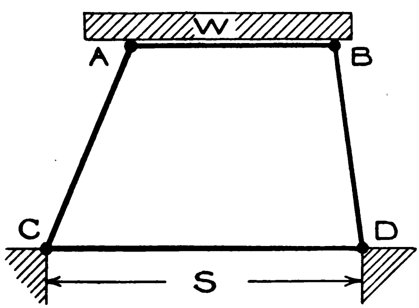


Fig. 53.

Fig. 53 shows a frame composed of four members, and it is obvious that such a contrivance could be readily adapted to carry a load W across the space S. Such an arrangement is, however, open to this

objection, that, should any lateral force, for instance due to wind, be applied to it, the members BD and AC would tend to rotate about their bottom pins D and C, and would consequently displace the load. To render the frame suitable for its work under these conditions we must therefore adopt some means of preventing this motion, we must prevent this deformation by stiffening the joints in some way and making the frame stable under all loads.

In practice, the frames used in roofs and bridges do not fulfil the conditions theoretically required of a frame, owing to the different arrangement of the parts, and hence frames used in this way are known as *trusses*.

Types of Frames.—Frames may be divided roughly into three distinct classes—

(*a*) *Firm Frames.*
(*b*) *Deficient Frames.*
(*c*) *Redundant Frames.*

Firm Frames.—An example of a firm frame is shown in Fig. 54 (*a*). It will be noticed that such a frame possesses just sufficient members to prevent any appreciable deformation under any load in the plane of the frame, provided the members are not stressed beyond their limit of safety. It should also be noticed that any one member may be lengthened or shortened without, in any way, affecting the stresses in the other two members.

Such a frame is stable for all loads within the breaking load.

Deficient Frames.—An example of a deficient frame is shown in Fig. 54 (*b*). Such a frame is not

possessed of sufficient members to prevent deformation on the application of a load. If we refer to our remarks on the funicular polygon we will readily see that there will be, at least, one system of forces which will keep the members of the frame in the given shape, but should one of these forces be altered, the effect would be such that the frame, as presently outlined by the members, would no longer coincide with the link or funicular polygon, and hence an alteration in shape would occur. Such a frame would

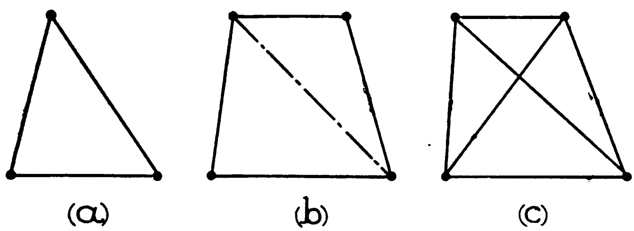


Fig. 54.

therefore not be stable under all loads, but it could be made so by the application of another member, as shown dotted. Such an addition would convert it into a firm frame, possessing stability under any system of loading, in the plane of the frame, and also possessing the further advantage that any one member may yet be lengthened or shortened without in any way affecting the stresses in the others.

Redundant Frames.—An example of this type is shown in Fig. 54 (*c*). A redundant frame may possess one or more members more than is necessary for stability, but it is stable under all loads, although

possessed of this disadvantage, that any alteration in the length of one member affects the stresses in all the others. Such a frame would therefore be affected by bad workmanship, and carelessness in marking off the lengths of the various members would result in initial stressing when the frame was fitted together. Such frames are sometimes spoken of as *self-strained frames.*

If n = number of joints in a frame, then a perfect frame should have $(2n-3)$ members.

Although roof and bridge trusses do not always

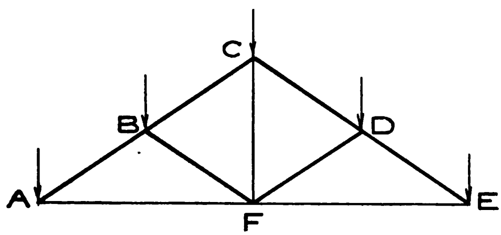


Fig. 55.

meet the theoretical requirements of a frame, yet it is found advantageous to assume that, in the generality of cases, the theoretical conditions are satisfied. In the truss shown in Fig. 55, for example, the rafters AC and CE are all in one piece, instead of being jointed at B and D respectively, while the tie-rod, instead of being formed of two separate members AF and FE, is all in one length. The joints B, C, D and F would be securely bolted or riveted, but, for our purpose, we assume that pin-joints exist at these points, and that AB, BC and such parts are all separate members. Such assumptions admit of the frame adjust-

ing itself, when loaded, so that all members are stressed axially. Pin-jointed structures are more in use in America than in this country, where the securely riveted or bolted joint finds most favour. Rigid joints give rise to bending stresses when the load comes on the structure, but, generally speaking, unless the structure be a very important one, such stresses are neglected.

Definitions.—The primary duty of a roof truss is to provide some means of support for the roofing materials as used to exclude the weather from a building. Trusses may be composed entirely of wood or iron, or they may be combined of both materials. A few terms in common use in connection with roof trusses may be worthy of short explanations.

Roof Truss, or *Principal*, is the name given to the framed structure complete without any roofing material.

Pitch of the Principals is the horizontal distance P between the centre lines of the trusses, as indicated in the plan in Fig. 56.

Rafter is the name given to the parts marked R_1, R_2, etc., extending from the abutment to the apex and providing support for the purlins. In the majority of cases the rafters are in compression.

Purlins are the cross roof-bearers which are attached to the rafters, the attachment being usually made by means of small pieces of angle-iron, known as *cleats*.

Struts and Ties.—Strut is the name given to all members subjected to a compressive stress, while all members in tension are known as ties.

Pitch is the slope of the rafter measured in degrees

from the horizontal. It depends on the nature of the material used in the covering. A table of weights of various roofing materials, together with minimum pitches, is given later.

Distribution of Loading.—In dealing with the dead loads acting on a roof truss these are, in general, taken as acting vertically at the joints, and the method of computing the magnitude of the loads at these various points will now be considered.

Let us consider the roof as shown in plan and elevation in Fig. 56. As shown by the shaded area each truss, or principal, carries a piece of the roof extending $\frac{P}{2}$ on either side of the centre line, so that the total width of roof carried by one principal is P, and if the length of each rafter be L ft., then the total area of roof supported by one principal will be $(2L \times P)$ sq. ft. Assuming the roofing material to weigh w lbs. per sq. ft., then the total load carried is obviously $(2L \times P \times w)$ lbs., which, for convenience, we can indicate as W. Now if this weight be uniformly distributed over the roof, we can take it that each section of the rafter gets an equal share of W. We have four rafter members, R_1, R_2, R_3, R_4, and hence each one carries a load of $\frac{W}{4}$ lbs., which we can assume either acts through its centre of gravity or is uniformly distributed along the member. This load, acting on say R_1, gives rise to reactions at 1 and 2 each equal to $\frac{W}{8}$; similarly, the load on R_2 gives rise

to reactions at 2 and 3, each equal to $\frac{W}{8}$, and so on right round. It will be observed that the joints over

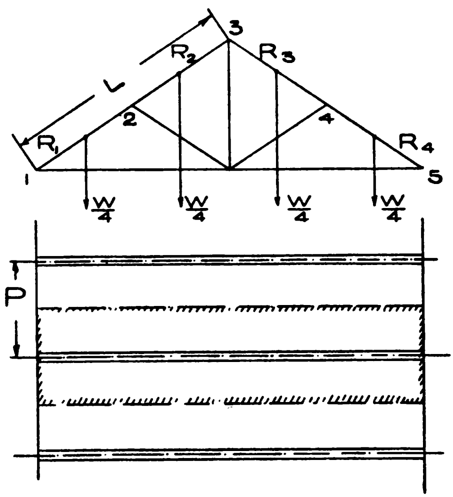


Fig. 56.

the abutments only get one part, $\left(\frac{W}{8}\right)$, while intermediate points get two parts, $\left(2\times\frac{W}{8}\right)$ or $\frac{W}{4}$. This can be stated in more general form, thus—

Let N = number of rafter members or parts.

W = total load on truss.

Load on joints over abutments $= \frac{W}{2N}$.

Load on intermediate joints $= \frac{W}{N}$.

In dealing with the loads on a roof truss it must not be forgotten that the truss itself may add very considerably to the dead load, and hence it is necessary that we have some means of approximating to the weight of any roof truss before the design is got out. An approximate formula is given below—

Let l = span in feet,
P = pitch of principals in feet,
W_1 = approximate weight in lbs.

$$W_1 = \tfrac{3}{4} lP\left(1 + \frac{l}{10}\right).$$

The following table gives the weight per square foot of different roofing materials, together with the minimum pitch at which they may be laid.

Material.	Weight (lbs.)	Pitch.
Tarred felt	·5	4°
Zinc	1·5-2	4°
Lead	5·5-8·5	4°
Corrugated iron, 16 G.	3·5	4°
Slates	6-10	26·5°
Tiles	10-16	30°
Boarding, $\tfrac{3}{4}$ in. thick	2·5	26·5°
Boarding, 1 in. thick	3·5	26·5°
Plate glazing	5	4°

The following simple example, fully worked out, may lead to a better understanding of the foregoing.

Example.—*A roof truss, of the type shown in Fig.* 55, *has a span of* 30 *ft., the rafters being* 16 *ft. long.*

The pitch of the principals is 10 *ft., and the roof is covered with* $\frac{3}{4}$ *in. boarding, and slates weighing* 7·0 *lbs. per sq. ft. Determine what load comes on each joint.*

Our first step is to find the approximate weight of the truss itself, and this is given by the formula given above—

$$\begin{aligned} W_1 &= \tfrac{3}{4} l P \left(1 + \frac{l}{10}\right) \\ &= \tfrac{3}{4} \times 30 \times 10 \left(1 + \frac{30}{10}\right) \\ &= \tfrac{3}{4} \times 30 \times 10 \times 4 \\ &= 900 \text{ lbs.} \end{aligned}$$

We must now determine the total weight of roofing material per sq. ft.

Boarding, $\frac{3}{4}$ in. thick, weighs 2·5 lbs. per sq. ft., slates weigh 7·0 lbs. per sq. ft., and purlins 1·5 lbs. per sq. ft.

Hence the total weight of roofing material per sq. ft. equals $7{\cdot}0 + 2{\cdot}5 + 1{\cdot}5 = 11$ lbs.

$$\begin{aligned} \text{Now } W &= w \times 2L \times P \\ &= 11 \times 2 \times 16 \times 10 \\ &= 3520 \text{ lbs.} \end{aligned}$$

$$\begin{aligned} \therefore \text{ Total weight carried} &= W_1 + W \\ \text{by one truss} &= 900 + 3520 \\ &= 4420 \text{ lbs.} \end{aligned}$$

This load is now divided up among the joints, apportioning one part each to A and E, and two parts

each to B, C and D; or, in other words, the load must be divided into eight parts, hence—

$$\text{Each part} = \frac{4420}{8} = 553 \text{ lbs. (say)}$$

$\therefore$ Load at A and E = 553 lbs. each.

$\therefore$ Load at B, C, D = 1106 lbs. each.

Simplest Form of Roof.—With the exception, perhaps, of the plain rafter as used in a lean-to roof, the type shown in Fig. 57 is about the simplest form with which we have to deal. The rafters are fastened together at their top ends, and the bottom ends are prevented from spreading by abutments on the supporting walls, so designed as to be able to withstand the lateral thrust of the rafters.

The first operation is to set down, to some suitable scale, the three joint-loads acting vertically downwards on the roof AB, BC and CD, in the line *abcd*, which is in reality the unclosed force polygon for these three forces. In dealing with problems such as the one above, it is advisable to adopt some convention regarding the treatment of the various joints, and the method adopted is to go round each joint in a clockwise direction, each joint at a time, starting from the left-hand side, and so form the polygon of forces for each joint.

Starting then at the left-hand abutment, we are at once beset with a difficulty, inasmuch as the reactions are unknown in either magnitude or direction. Passing on, then, to the apex point, we find there are three forces, of which one is known in magnitude

and direction, while the directions of the other two are known.

Going round this point clockwise, and starting with our known force BC, we find in our force

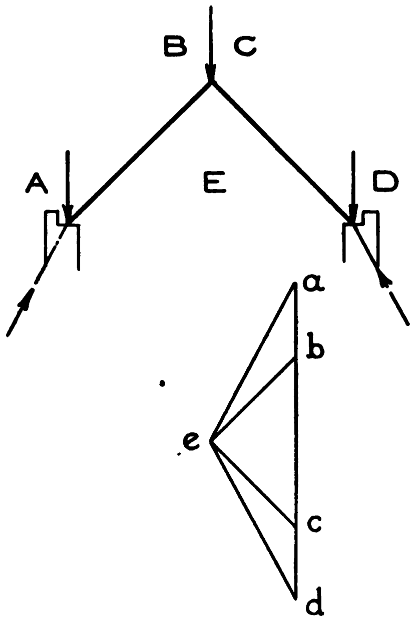


Fig. 57.

polygon the line *bc* representing it; the next force in order is CE, and from *c* we now draw a line *ce* parallel to the member CE, while the third force is EB, and therefore we draw, from *b*, a line *be* parallel to EB (we draw from *b* in this instance, because, as yet, the point *e* is not fixed). Join *ea* and *ed*. Then

eb and *ec* represent the stresses in EB and EC respectively, while *ea* and *ed* represent the magnitude and direction of the reactions.

Simple Roof Truss with Single Tie-rod.—Fig. 58 shows the previous type of roof modified so that the

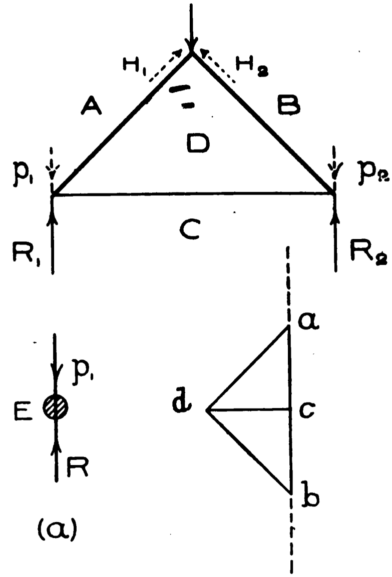


Fig. 58.

outward thrust of the rafters is taken by a tie-rod CD instead of by the abutments, thereby making the frame self-contained. In this case, since the reactions have no lateral components, they will be perpendicular, and the effect of this on the loads p_1 and p_2 is worth noticing. Considering any one of these points, we have

a state of matters as shown in Fig. 58 (a). We have a point E acted on by a force R, which is the total reaction at that point caused by the load on the truss; and also by a force p_1, in the same straight line but in the opposite direction, so that the net force acting on A is obviously $(R - p_1)$, which force we have called R_1. The point to be noted is this, that, in a case where the reactions are perpendicular, the vertical loads acting at the joints just over the abutments, do not, in any way, affect the stress in the various members, and may therefore be rejected, but it should also be carefully noted that, as far as the design of the supporting walls or columns is concerned, this value R_1 must not be used, but the true value of the reaction R, which equals $(R_1 + p_1)$.

In lettering our diagram then, we need consider only the apex load AB. The construction of the stress diagram does not call for much further explanation. The load AB is represented by the line *ab* drawn parallel and equal to it. From *b* we draw *bd* parallel to BD, and from *a* we draw *ad* parallel to AD. The force in the tie-rod is settled by going round one of the reaction-points—say R_1. We have *ad* parallel to AD, and our next force, going round clockwise, is DC, hence we draw from *d* a line *dc* parallel to the tie-rod. The lines *bd* and *da* give the loads in the rafters; *cd* gives the load on the tie-rod; while *bc* and *ca* give the values of the reactions R_2 and R_1 respectively.

In a simple case like this, it is an easy matter to discriminate between ties and struts, but, in more

complicated structures, some special method must be adopted whereby we can determine the nature of the stress in any of the members composing the structure, and if the following method of procedure be adopted, no difficulty should be encountered.

At the apex point we have three forces acting along the three given directions, and forming a simple system in equilibrium. The three forces will be represented in our force diagram by a triangle *abd*, whose sides *taken in order* will give us a clue to the nature of the several forces. We know the sense of only one force at the apex-point, and that is *downwards* towards the point, hence in our force diagram the lettering *ab* is in a *downward* direction, so that we pass down from *a* to *b*, and then in order from *b* to *d*, indicating that the force in BD acts from right to left, or in the direction indicated by the dotted arrow H_2. Similarly with the force in DA, it acts in the direction of the arrow H_1. Both BD and DA are pushing on the point at the apex, and we would find that they were each also pushing on the points at the reactions. We have therefore two divergent arrowheads on each member, similar to that shown in Fig. 59 (*a*), and hence the stress is a compressive one. Should the arrowheads be convergent, as shown in Fig. 59 (*b*), then the member is in tension.

Fig. 59.

At first sight it would seem that, if the arrowheads pointed outwards, as at (*a*), we had a decided indica-

tion of tension, but any difficulty in the matter will be easily overcome if we bear in mind that the arrowheads do not indicate the forces acting on the member, but serve as an indication of the manner in which the member itself acts on the joints at its extremities. If therefore a member acts *outwards* on two joints, then these joints must act *inwards* on the member, and consequently it will be in compression.

The arrowheads on CO will be convergent, indicating tension, as is to be expected.

It is well to cultivate some system and order in tabulating results, and the following method, as indicated by Fig. 60, is, perhaps, as good as any.

MEMBER	BF	CG	EF	FG	CH	HE	
COMPRESS^N	6·1	5·8	-	0·8	-	-	TONS
TENSION	-	-	5·4	-	3·2	2·9	TONS

Fig. 60.

Simple King-rod Truss.—This truss is a simple modification of the truss shown in the preceding example, the addition being a light king-rod from the apex to the centre of length of the horizontal tie-rod. It will be found that the stress in this king-rod is zero, showing it to be a redundant member, whose only duty is to assist in supporting the horizontal tie.

The rise is, in general, about one-fifth of the span, and, since the rafters are unsupported at intermediate points, this construction is unsuited for spans exceeding 20 feet.

Swiss Roof Truss.—A further modification of the above, allowing a little more headroom in the centre of the building, is shown in Fig. 62. It will be seen from the stress diagram that the stress in the king-rod is no longer zero, but has an amount equivalent to *fg* in the force diagram. To construct the force diagram we begin by setting down the loads AB, BC, CD (*N.B.*—AB and CD may be neglected) in the line *abcd*. Now, since the loading is symmetrical, the reactions will be each equal to one another, so

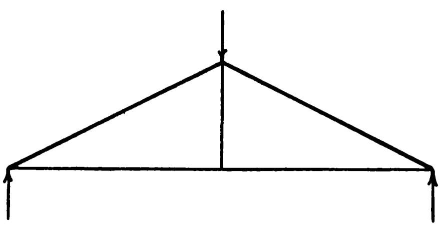


Fig. 61.

that, if we divide *ad* in *e*, the two lines *ae* and *ed* will give the magnitude of the two total reactions.

Considering now the left-hand abutment, and going round the point clockwise, we have *ea* up, parallel and equal to the reaction EA; then, following this, we have *ab* down; the next force now is the one acting along BF, and from *b* in the force diagram we draw *bf* parallel to BF, in the position diagram. The only remaining force acting at the point is that along FE, and from *e* we draw a line *ef* parallel to EF to cut *bf* in *f*. Take now the apex-point and follow the forces round clockwise, and lastly

the right-hand abutment, and no difficulty should be experienced in completing the force diagram

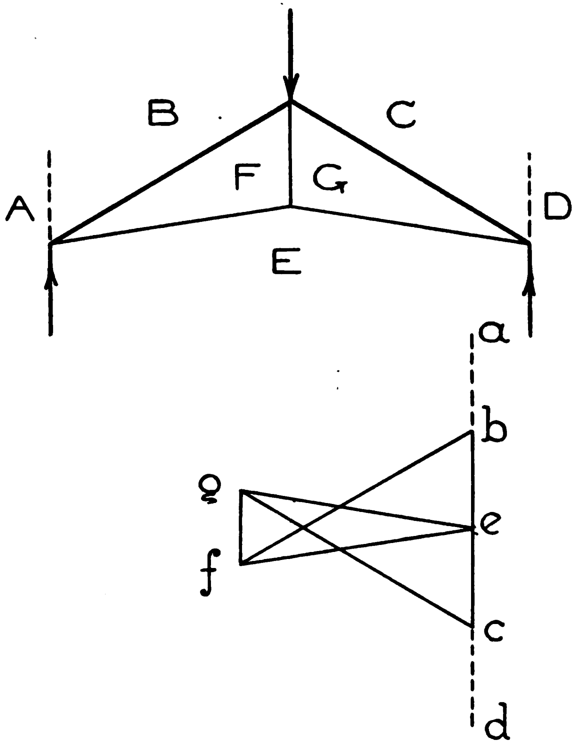


Fig. 62.

as shown. The stresses can then be scaled off and tabulated.

This type is suitable for spans up to 20 ft. The rise is usually about one-fifth of the span, and

the rise of the tie-rod about one-thirtieth of the span.

Compound Swiss Truss.—For larger spans, the

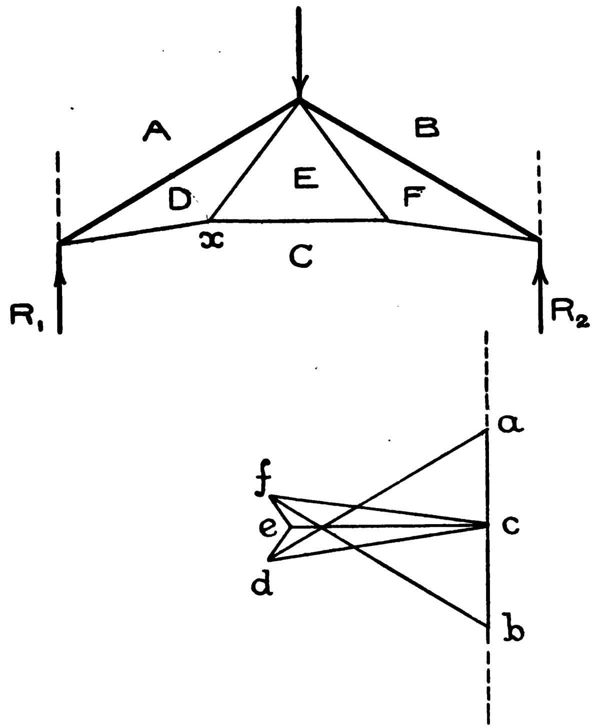


Fig. 63.

outline of the previous truss may be modified, as shown in Fig. 63, the truss now being known as the compound Swiss truss. The method of solution should be apparent from the figure. If, however, the

apex-point be taken just after the point over R_1, it will be found that a difficulty arises, due to the fact that the point *e* cannot be determined. It will be readily seen that the solution is indeterminate, since we have three unknown magnitudes to deal with.

The difficulty can be got over by treating the point *x* after R_1 and by so doing getting the force triangle *cde;* this makes it possible to construct the force polygon *abfed* for the apex-point. It should be noted that, in completing the force diagram, the line *cf* forms a check line, for the points *c* and *f* are fixed, and consequently the line *cf* must coincide with a line through *c* parallel to CF.

Compound Swiss Truss with Right-angled Struts.—The struts are added to support the rafters at their mid-points. They are at right angles to the rafter, and on that account this truss is sometimes known as the right-angled strut truss.

In drawing out the force diagram we are again met with the difficulty experienced in the previous case, but solution is possible by first treating the point *x*, before passing on to the apex-point. Another solution, though not strictly graphical, might easily be adopted. We can fix the point *h* as soon as we know the value of the tension in HE, and this can be got very readily in the following manner:—

Let T = tension in HE,

y = perpendicular distance from apex to HE,

$2S$ = span,

W = load at each joint.

Taking moments about apex we have—

$$R_1 \times S = \left(W \times \frac{S}{2}\right) + (T \times y),$$

$$\therefore T = \frac{(R_1 \times S) - \left(W \times \frac{S}{2}\right)}{y} = \frac{S}{y}\left(R_1 - \frac{W}{2}\right).$$

This tension can now be marked off in the force

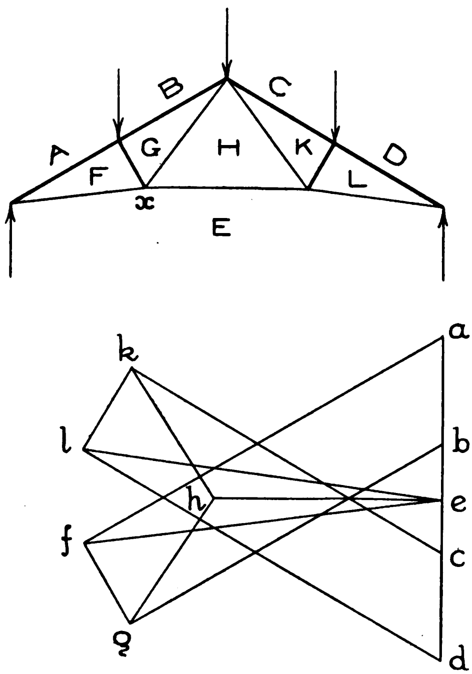


Fig. 64.

diagram in the line *eh*, the point *h* being fixed in this manner.

The above type is one of the earliest types of iron roofs, and is probably the best and simplest for small spans up to 30 ft. Its outstanding advantage lies in the fact that all the bracing is in tension, and it has

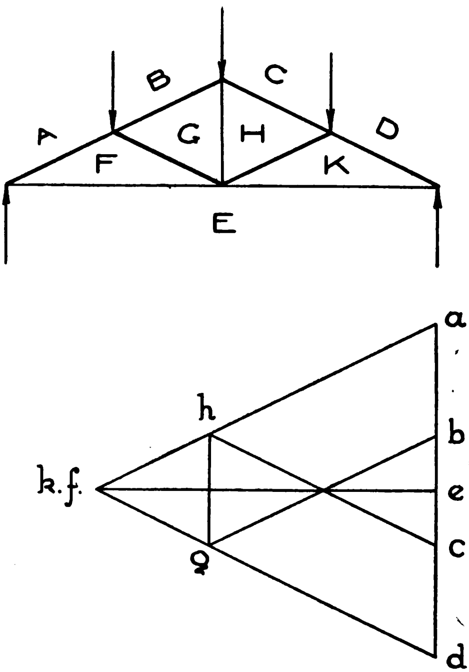


Fig. 65.

been shown in practice that, if the members be designed in proportion to the stress, this type of roof is one of the most economical in material. The rise is about one-fifth of the span.

Simple King-rod Truss with Single Struts.—

The addition of the struts allows the king-rod truss to be used for spans up to 30 ft.

In the case shown in the figure, the loading is again taken as symmetrical, and hence the reactions R_1 and R_2 are each equal to $\frac{1}{2}$(AB+BC+CD), and are represented, in the force diagram, by the lines *ea* and *de*. The only points to be noticed are that the points *k* and *f* in the force diagram are common, and that in going round the joint under the load CD, the member KH forms a check line in force polygon.

King-rod Truss with Struts and Inclined Ties.—In order to gain a little more headroom in the centre of the floor, the previous type of truss is sometimes modified, and the lower ties inclined, in place of being horizontal. The method of solution will be readily followed from the diagram, and it should be noted that *k* and *f* are no longer common points. When this type of roof is used for spans over 30 ft., it is usual to fit additional suspension rods, as shown dotted.

Swiss Truss with Two Struts or Belgian Truss.—Fig. 67 shows a further modification of the Swiss truss, by the addition of two struts to support the rafter.

In setting out the truss, the rafter is divided into three equal parts. The height of the tie-rod is then settled, and the perpendicular bisector of the rafter drawn to cut the line of the tie-rod in x; from this point x the struts are drawn to the points of trisection of the rafters. The drawing of the force diagram

presents no difficulty, and should be readily followed from the diagram.

This type of truss is frequently made without any

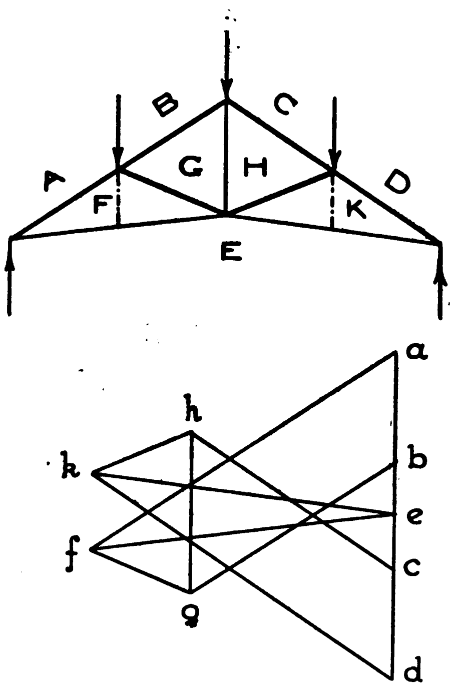


Fig. 66.

rise in the tie-rod, and in such a case the struts KL and NO are vertical. A simple and inexpensive roof of this type, in which the rafters and struts are made of wood, was first introduced by Mr H. P. Holt, C.E.

This type can be designed for spans ranging from 20 to 50 ft. The standard pitch of the principals is 10 ft., and the principals are tied together by the

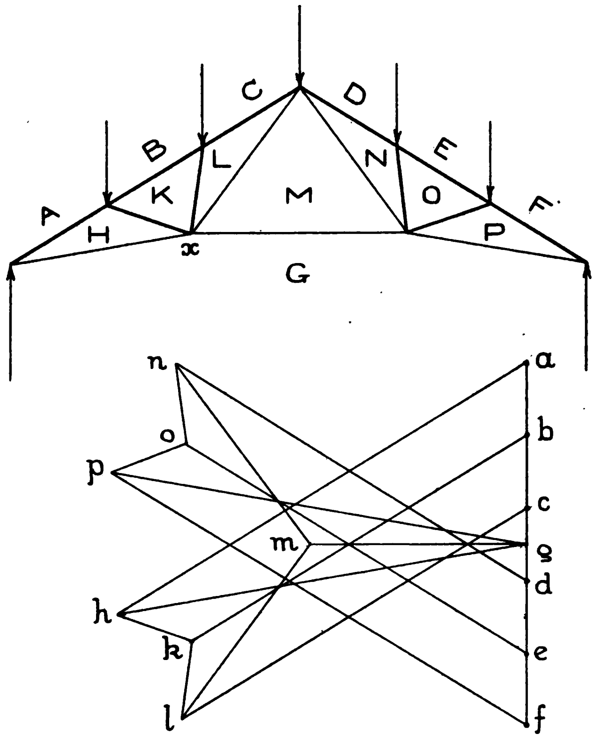


Fig. 67.

purlins, which, being wood of a substantial section, are notched on to the rafters.

English Truss.—The English truss is a king-rod truss with three struts, and is used for spans up to

70 ft. The drawing of the stress diagram will present no difficulty, but, in drawing out the frame diagram, the members KL and WX might well be dispensed

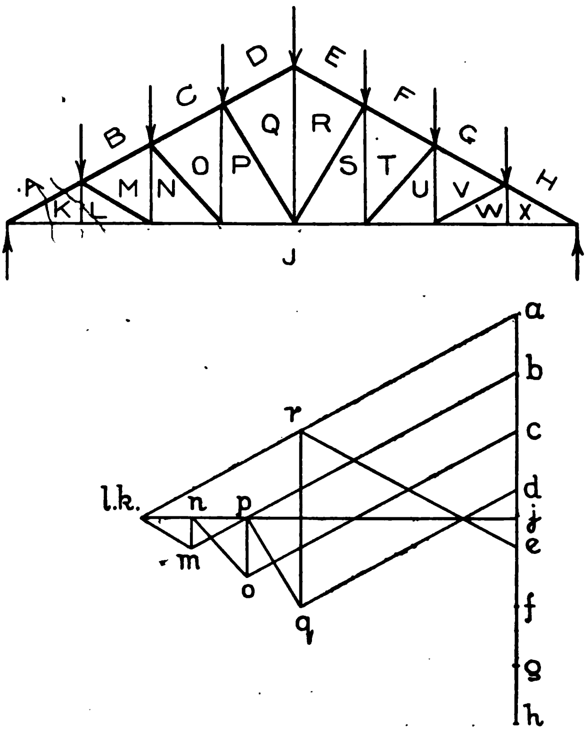


Fig. 68.

with, as no stress is induced in them by the external loads, their sole purpose being to support the long sections of the lower tie-rod.

Simple Queen-post Roof.—Fig. 69 shows a type of

truss known as the queen-post truss, the members X, X being known as *queen-posts*. As constructed in wood, this forms a very good type of roof for spans up to 35 ft. It will be noticed, from the line diagram superposed on the truss, that the structure is composed of two distinct types of frames, the centre portion being a *deficient frame* and the two side wings *firm frames*. Such a structure, theoretically considered, would, under an unsymmetrical system of loading, be deformed, but in practice the wooden crossbeam

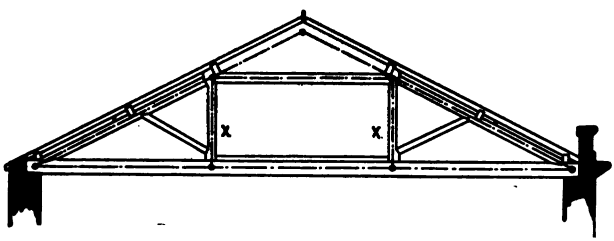


Fig. 69.

forming the tie, is usually of such massive proportions as to afford sufficient rigidity to prevent deformation. In the iron truss, however, this rigidity cannot be obtained from the iron tie-rod, and hence some means of counterbracing the central portion must be adopted. The simple queen-post truss, as shown in Fig. 70, shows one modification of this truss to adapt it to the requirements of iron construction, for spans up to 30 ft.

The difficulty with indeterminate forces occurs when we come to consider the apex-point, but this

difficulty can be got over by considering the joint marked x after that under the load AB, or by treating the point counterclockwise. The complete solution will be easily followed from the diagram.

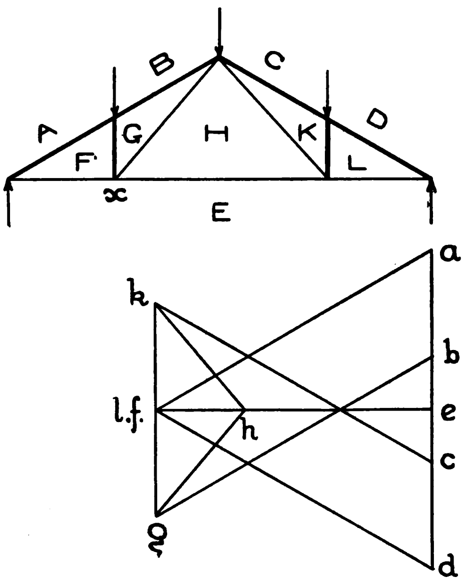


Fig. 70.

Compound Queen-post.—From a comparison of Figs. 68 and 71, it will be noticed that this truss has a very close resemblance to the *English truss*, but, by a little study, the following important distinctions will be readily noted. The uprights are struts, and the inclined members are ties, while in the English truss the opposite order of things holds good, and

also the inclined members in the queen-post truss slope from the bottom inwards, while those in the English truss slope from the bottom outwards.

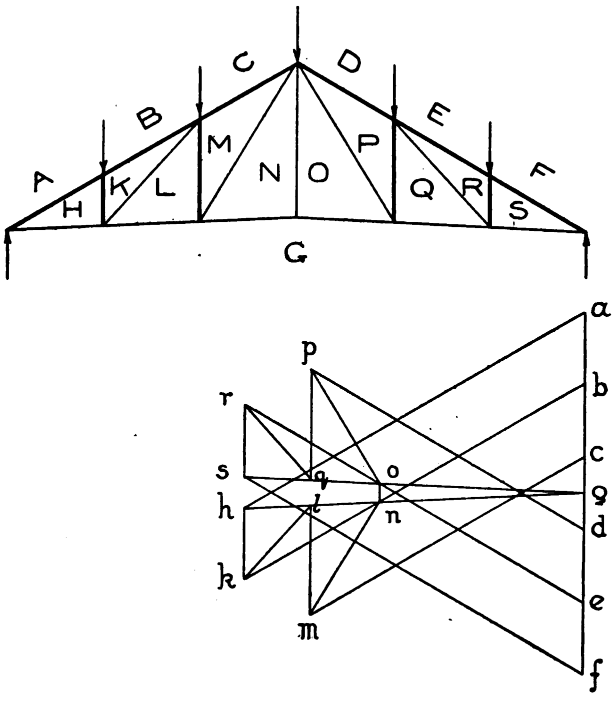


Fig. 71.

The force diagram, worked out alongside, gives a ready clue to the complete solution. Many modifications of this type exist, but all lend themselves to easy solution along the lines indicated.

French Roof Truss.—Fig. 72 shows a very common

type of truss, known as the French truss, as commonly used for spans from 45 to 60 ft. The drawing

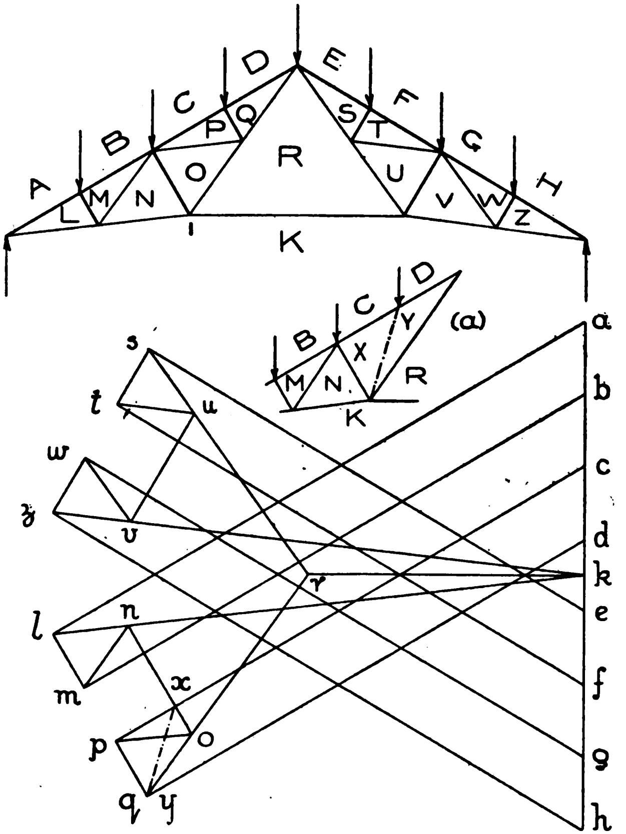


Fig. 72.

of the force diagram presents no difficulty until the joint under the load BC is reached. To all appear-

ances, the solution is indeterminate, for, of the six forces acting on the point, we know the magnitude and direction of only two, while, of the other four, we know only their directions.

In overcoming this difficulty, *three methods of solution* are open to us.

(*a*) We can assume that the stresses in MN and OP are equal. (This, however, is only true when the loading is symmetrical.) By this means we can fix the point *p*, by the aid of simple geometry, and no further difficulty will be met in that half of the frame.

(*b*) We can fix the point *r*, by calculating the stress in the horizontal tie-rod KR, as explained in a previous example, by taking moments about the apex and considering the equilibrium of one-half of the truss.

(*c*) We can make use of the substitution frame. This method, first introduced by Professor Barr, is by far the most satisfactory method.

The first two methods do not call for any further explanation, but the third is worthy of a little further study.

As shown in Fig. 72 (*a*), the two members, OP and PQ, are omitted, and, in their stead, a substitution member, XY, is introduced. If we now go round the point, we have *bc* parallel to BC, from *c* we draw *cx* parallel to CX, and *xn* can now be drawn parallel to XN, the point *n* having previously been fixed by going round the joint at the foot of the strut LM. We can now go round the point under CD, the lines forming the force polygon being *cd*, *dy*, *yx*, and finally *xc*. We now

pass to the lower joint marked 1, and form the force polygon *knxyrk*. The members, OP and PQ, are now replaced, and the forces acting at the joints are now treated in the usual way, treating joint marked 1 before those under BC and CD. No further difficulty will be encountered until the joint under FG is

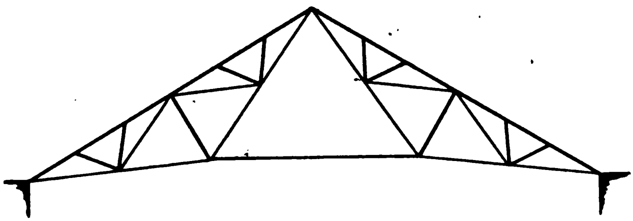


Fig. 73.

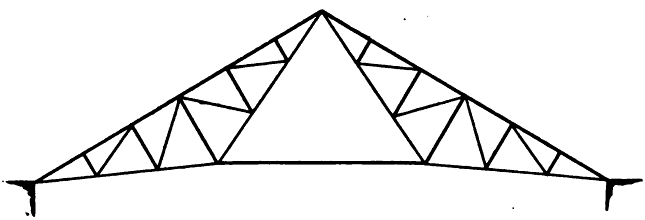


Fig. 74.

reached, but the difficulty is got over by making use of the substitution member as before. The modifications of the French roof truss, as shown in Figs. 73 and 74, are adopted when this type of roof is used for spans over 60 ft.

The student who has conscientiously worked through all the preceding examples should now be in a position to solve the above cases for himself.

Mansard Roof.—Like the queen-post truss, this type of truss, when made of iron, although retaining the same outline, differs considerably in the arrangement of its members from the wooden truss which bears the same name. Fig. 75 shows the Mansard roof as constructed of timber, while Fig. 76 shows the modified arrangement of members when constructed of iron.

The difficulty of seemingly indeterminate forces

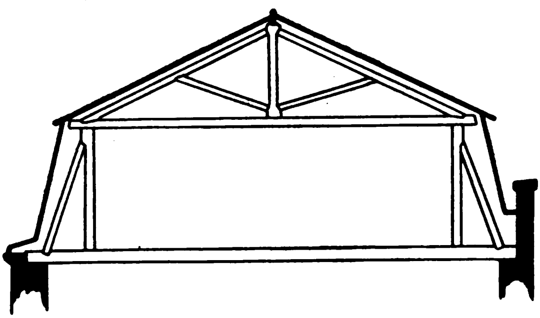


Fig. 75.

will again be met with at the joint under the load AB, but the student should have now no trouble in overcoming such difficulties.

Other modifications of the Mansard roof are given in Figs. 77 to 80, but it is left to the student to solve these for himself.

Examples.

1. A simple roof truss consists of two rafters and a horizontal tie-rod connecting the ends which rest on the abutments. The span is 18 ft. and the rise 4 ft.

The roof covering is equivalent to a dead load of 660

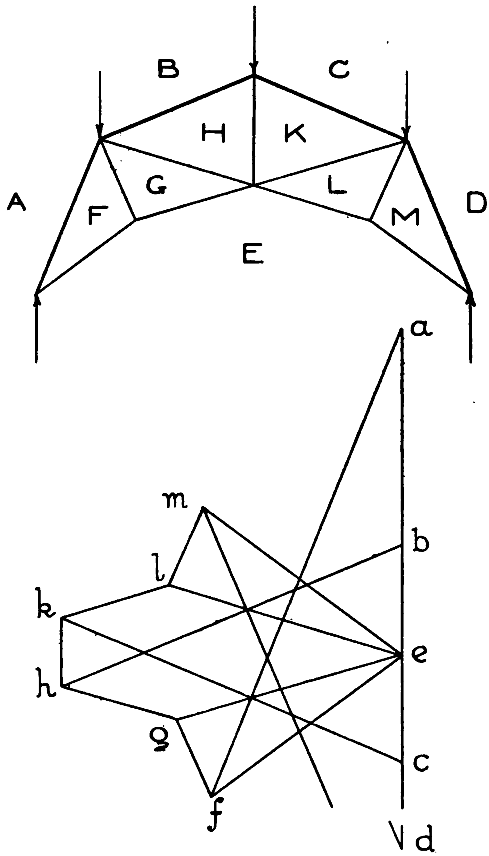


Fig. 76.

lbs. at the apex. Determine the thrust in the rafters and the pull in the tie-rod.

2. A Swiss roof truss is used for a span of 20 ft. The rise to the apex is 4·5 ft. and the lower tie-rods rise to a point 9 ins. above the level of the abutments. The load on the apex is 800 lbs. Determine and tabulate the stresses in the various members.

3. A storage shed is roofed in with boarding and slates carried on compound Swiss trusses. The span is 24 ft. The rise to the apex is 6 ft., and the lower tie-

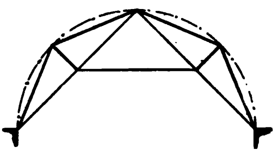


Fig. 77.

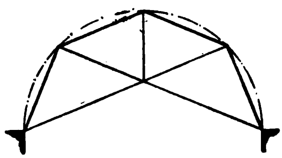


Fig. 78.

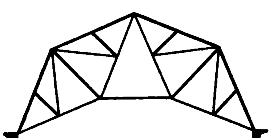


Fig. 79.

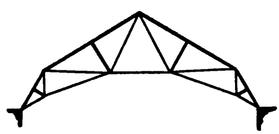


Fig. 80.

rod has a rise of 1 ft. The pitch of the principals is 5 ft. The roofing boards are $\frac{3}{4}$ in. thick and the slates weigh 8 lbs. per square ft. Determine and tabulate the stresses in the members.

4. A small workshop is to be covered in with plate-glass. The span is 30 ft., and it has been decided to use a compound Swiss truss with right-angled struts. The rise to the apex is 6·5 ft. and to the tie-rod 1 ft. The pitch of the principals is 6 ft. Plate-glass weighs

5 lbs. per sq. ft., and the supports for the glass can be taken as $1\frac{1}{2}$ lbs. per sq. ft. Determine the stresses in the members.

5. A Belgian truss is used in roofing over an engine shed. The span is 40 ft. and the rise to the apex 8 ft. The lower tie-rod has a rise of 18 ins. The pitch of the principals is 10 ft. Boarding, 1 in. thick, is used to carry slates weighing 8 lbs. per sq. ft. The purlins and fastenings can be taken as $1\frac{1}{4}$ lbs. per sq. ft. Take into account the weight of the truss. Determine the stresses in the various members, distinguishing carefully between ties and struts.

6. The shed in the above question was altered to give a span of 60 ft., and English trusses were adopted for the roof, the pitch of the principals being kept as before. The height to the apex was 14 ft., and the lower tie-rods rose to a point 2 ft. above the level of the abutments. Determine the stresses in the members, the roofing material being of the same weight per sq. foot as before.

7. A compound queen-post truss carries a roof over a span of 45 ft. The lower tie-rod is horizontal, and the rise of the apex is 10 ft. The loads over the abutments are each 500 lbs., and at the other joints 1000 lbs. Determine and tabulate the stresses.

8. A workshop roof is carried on French-roof trusses. The span is 40 ft. and the rise to the apex 8 ft. The lower tie-rod has a rise of 18 ins., the pitch of the principals being 8 ft. The roofing material is No. 16 G. corrugated iron, weighing 3·5 lbs. per sq. ft. The purlins and bearers can be taken

as 1·5 per sq. ft. Taking into account the weight of the truss, determine the stresses in the various members.

9. A Mansard roof truss, as shown in Fig. 77, is used for a roof having a span of 40 ft. The loads over the abutments are each 600 lbs., and at each of the other joints 1200 lbs. Determine the stresses induced in the various members of the truss due to these loads.

10. Assuming the same loading as given in Question 9, determine the stresses in the various members of the roof trusses shown in Figs. 78, 79, 80. The span in each case can be taken as 50 ft., and the proportions transferred from the diagrams.

CHAPTER VIII

ROOFS (*continued*)

Unsymmetrical Roofs.—All the roof trusses with which we have dealt so far have been symmetrically loaded about their centre-lines, and hence the finding of the reactions has given rise to no difficulty. However, in dealing with unsymmetrical loadings, the case becomes a little more complicated, and some graphical method must be adopted for the determination of the values of the reactions. The following solution is applicable to all cases, whether the lack of symmetry be due simply to inequalities of the loadings at the various joints, or to inequalities in the spacing of the joints by reason of the construction of the roof.

We will now consider a few roofs of the latter type, and the method to be used in finding the reactions will be found to be applicable to other cases embodying unsymmetrical loadings.

Saw-tooth or Northern Lights Roof.—This type of roof, shown in outline in Fig. 81, takes its name from the fact that the shorter side is glazed and faces the north, thereby admitting light, though avoiding the direct glare of the sun.

Set down *ab* and *bc*, as shown, and select a pole *o*. Join *oa*, *ob* and *oc*. Produce the lines of action of

the loads and construct the funicular polygon *pqrs*, finishing with the closing line *sp*. From *o* draw *od*,

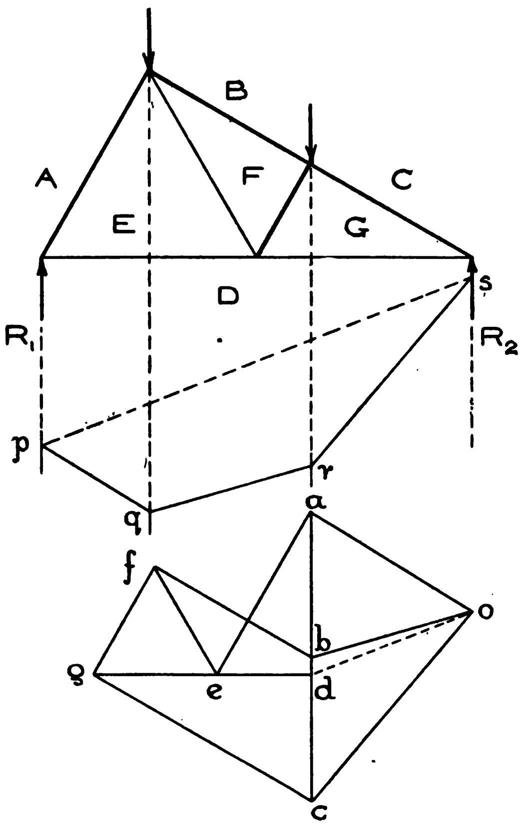


Fig. 81.

parallel to *sp*, then *da* and *cd* represent the reactions, R_1 and R_2, respectively. The drawing of the stress

diagram will now present no difficulty, and should be readily followed from the diagram.

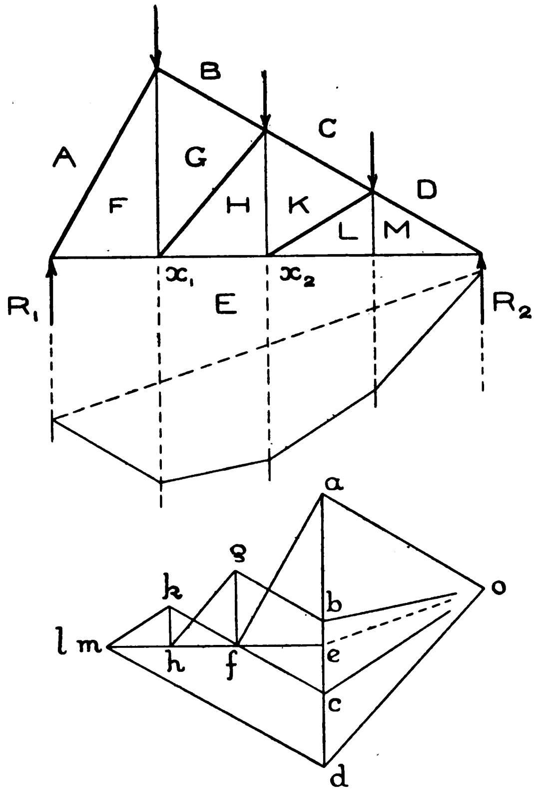


Fig. 82.

For wider spans it becomes necessary to stiffen the long rafter with additional struts, one modification appearing as in Fig. 82.

As before, the funicular polygon is made use of in determining the reactions R_1 and R_2. In drawing out the stress diagram, difficulty will be experienced at the joints under the loads BC and CD, the forces acting at these points being seemingly indeterminate. The difficulty can be got over by treating the joints

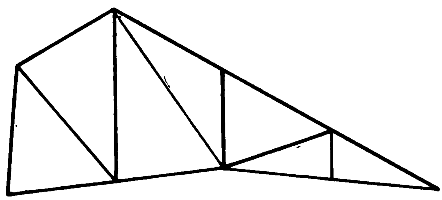


Fig. 83.

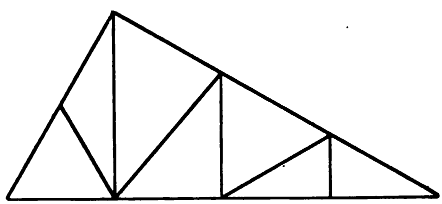


Fig. 84.

in the following order, viz., R_1, AB, x_1, BC, x_2, CD and R_2. The difficulty at BC can also be overcome by going round the joint counterclockwise, noting, at the same time, that the point *h* must fall on a horizontal through *e*, since *eh* is a horizontal member.

Figs. 83 and 84 show further modifications of this truss. The student is left to work out these cases for himself. The finding of the reactions and the

drawing of the stress diagrams should now present no difficulty.

Overhung Roof or Pent Truss.—In Fig. 85 is shown one type of pent truss carrying five loads on the upper joints. In this case, before we can obtain particulars of the reactions, R_1 and R_2, we must first

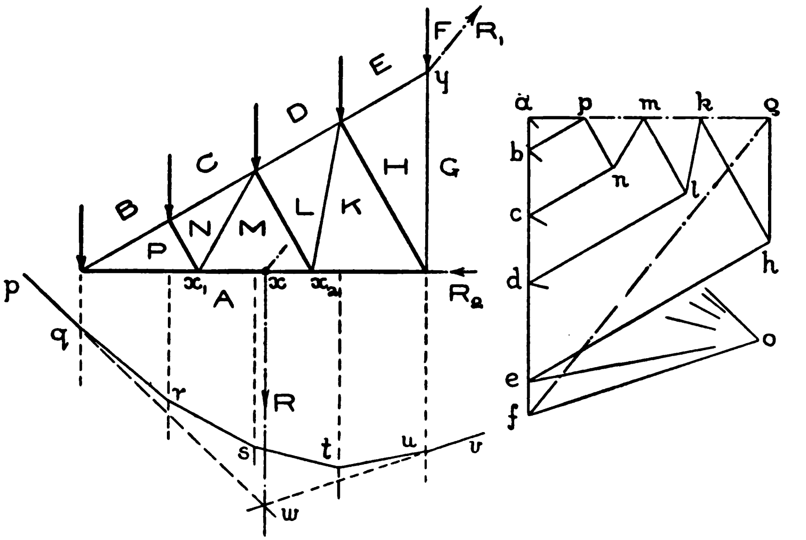


Fig. 85.

find the resultant of the external loads. As explained in Chapter IV., Fig. 30, construct the funicular polygon *pqrstuv*, and produce the first and last lines to intersect in *w*. Through *w* draw R parallel to *af*. Now the truss, as a whole, is acted on by three forces, which keep it in equilibrium ; hence the three forces, R, R_1 and R_2, must meet in a common point. The

point common to R and R_2 is x, so that a line through x and y will give the line of action of R_1. We can now determine the magnitude of R_1 and R_2 by drawing the triangle of forces, *afg*.

The stress diagram will present no difficulty if the

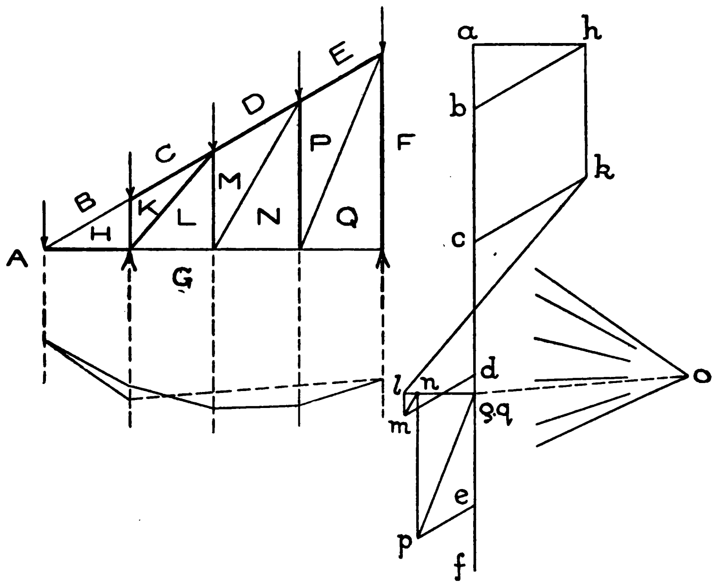


Fig. 86.

joints be taken in the following order, viz., AB, BC, x_1, CD, x_2, DE, EF, and R_2.

Overhung Roof with Pillar.—Fig. 86 shows a modification of the overhung roof, as shown in the preceding example. In this case, the roof, instead of being supported entirely at the wall, is further supported by a front pillar, so that, in reality, it is only

the extreme panel of the truss which is overhung. The reactions will be vertical in this case, and can be easily determined with the aid of the funicular polygon. It will be noticed that there is a change in the nature of the stress in the rafter members at the joint under the load BC, the member BH being in tension, while all the other rafter members are in

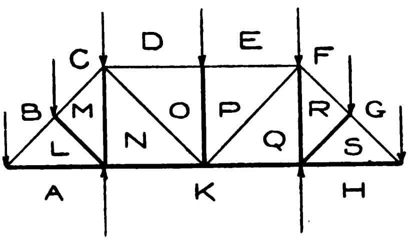
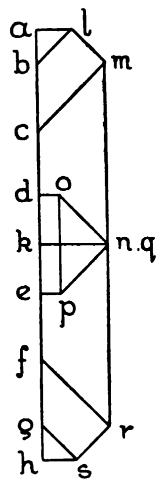


Fig. 87.



compression. On the under side, the member AH is in compression, while all the other lower horizontal members are in tension. The solution will be readily followed from the diagram in Fig. 86.

Island Station Roof.—Fig. 87 shows a type of roof frequently used for island stations. The case of simple loading, as shown, presents no difficulties, and will be easily followed from the diagram. When acted on by wind, the case becomes a little more complicated, but the solution of such a case will be given in the next chapter dealing with wind loads.

Examples.

1. A saw-tooth roof truss has the outline shown in Fig. 81. The apex angle is 90° and the abutment angles 60° and 30° respectively. The span is 22 ft., and the pitch of the principals is 8 ft. The member FG is the perpendicular bisector of the longer rafter. The longer side is covered with slates and boarding weighing 12 lbs. per sq. ft., while the shorter side is covered with glass weighing 5 lbs. per sq. ft. Determine and tabulate the stresses in the various members.

2. Draw out a roof truss similar to that shown in Fig. 82, the angles being 30°, 60°, and 90°. The span is 32 ft. and the pitch of the principals 10 ft. Assuming the same type of covering as in Question 1, determine and tabulate the stresses in the various members. Write down the value of the reactions R_1 and R_2.

3. Draw out a roof, as shown in Fig. 83, taking the proportions from the diagram. With same loading as in the above cases, determine the stress in the members, if the span of the roof is 40 ft. and the pitch of the principals 10 ft.

4. The roof of Question 3 was finally altered to the shape shown in Fig. 84, all other things remaining as before. Adopting the outline and proportions of Fig. 84, determine and write down the stresses in the members, and determine also the value of the reactions.

5. An overhung station roof is arranged, as shown in Fig. 85, the amount of overhang being 20 ft. The

rafter makes an angle of 30° with the horizontal, and is divided into four equal portions, being supported at the joints by struts normal to the rafter. The trusses are 8 ft. apart, and the roofing material weighs in all 14 lbs. per sq. ft. Determine the stresses in the various members and the magnitude and direction of the reactions.

6. Assuming the same truss as in Question 5, determine the stresses in the members and the reactions when the total load on each truss is apportioned in the following ratio—

AB = 1, BC = 2, CD = 3, DE = 3·5, and EF = 1·5.

7. Fig. 86 shows the type of roof adopted for a certain railway station. The overhang is 40 ft. and the pillar is 10 ft. from the outmost point. The trusses are 10 ft. apart, and the roofing material weighs 14·5 lbs. per sq. ft. Determine the stresses in the members and the magnitude of the reactions.

8. An island station roof has the outline shown in Fig. 87. The overall span is 40 ft. and the distance between the pillars is 20 ft. The depth of the truss is 10 ft. and the trusses are 10 ft. apart. Determine the stresses in the members if the roofing material weighs 12 lbs. per sq. ft.

CHAPTER IX

ROOF TRUSSES—WIND PRESSURE

Calculation of Wind Pressure.—So far, we have been dealing with the effects produced on structures by the application of dead loads only, but all structures exposed to the weather are, more or less, subjected to additional loads due to wind pressure, which may be so great, in some cases, as to cause the stresses induced by the dead loads to appear almost negligible in comparison to those caused by the wind. Many causes influence the pressure which the wind exerts on a structure, and a mathematical solution of the practical case is impossible. We can, however, obtain an expression for the force exerted by the wind, if we make certain assumptions. Let us consider a stream of air to impinge on a flat surface, whose area is greater than the area of the stream, and further let this surface be perpendicular to the direction of the air stream, then the pressure exerted on the surface can be deduced thus—

Let P = air pressure in lbs. per sq. ft.,
v = velocity of air in ft. per second,
W = weight of air delivered per sq. ft. p. sec. in lbs.,
w = weight of 1 cubic ft. of air in lbs.

After impinging on the surface the air is assumed to

have no velocity normal to the surface. We have, in general—

$$\text{Change of momentum} = \text{impulse}—$$

$$\frac{W}{g} \times v = P \times t.$$

In our case, we are dealing with one second, and hence t = unity; we have therefore this relationship—

$$P = \frac{W}{g} \times v,$$

or expressed in words—*the pressure in lbs. per sq. ft. is equal to the change of momentum per second occurring on that area.* Now we are dealing with 1 sq. ft. and an air velocity of v ft. per second, hence the air delivered per sq. ft. per second will be v cubic ft.

$$\therefore \text{ Weight of air delivered per second} = w \times v \text{ lbs.}$$
$$= W \text{ lbs.}$$

$$\text{Now change of momentum p. sec.} = \frac{W}{g} \times v.$$
$$= \frac{w \times v}{g} \times v.$$
$$= \frac{wv^2}{g}.$$

Putting in the values of w (=0·08 lbs.) and g (=32·2) and converting the velocity of the wind from ft. per second into miles per hour, we get—

$$\text{Change of momentum p. sec.} = \frac{\cdot 08 \times V^2 \times 5280 \times 5280}{3600 \times 3600 \times 32 \cdot 2}$$
$$= 0 \cdot 0053 V^2,$$

where V = air velocity in miles per hour.

$$\therefore P = 0 \cdot 0053 V^2 \text{ lbs. per sq. ft.}$$

Thus, with wind at a velocity of 40 miles per hour, representing a moderate gale, the pressure in lbs. per sq. ft. would be $0{\cdot}0053 \times 40 \times 40 = 8{\cdot}5$ lbs., assuming the above conditions could be attained in practice.

Experiments by Dr Stanton at the National Physical Laboratory go to show that the pressures, as obtained by the above formula, are much too high. These experiments show that for rectangular surfaces $P = 0{\cdot}0032V^2$ lbs. per sq. ft., no matter what area be exposed to the force of the wind. For lattice-work it was found that the pressure was much greater, and that for structures of this nature $P = 0{\cdot}00405V^2$ lbs. per. sq. ft.

The effect of suction on the leeward side has an important effect on the force which the wind exerts on any structure, and this suction effect is found to fall off rapidly as the length of the structure is increased in the direction of the wind. Lattice-work, presenting as it does innumerable thin, flat plates to the wind, suffers increased pressures due to this suction effect.

It can be taken generally that the steeper the slope of a roof the greater will be the suction effect on the leeward side, and hence the greater the effective force on the windward side. The experiments at the Forth Bridge showed the effect of the wind to increase with height above the ground. The shape of the surface and its position relative to other buildings are factors which very materially affect the force exerted by the wind.

Maximum Wind Pressure.—The maximum wind pressure allowed on structures in this country was for a long time taken as 40 lbs. per sq. ft., and if we make use of the theoretical formula which we obtained, and calculate the pressure exerted by a hurricane, which means a velocity of 90 miles per hour, we find the wind pressure to be about 41 lbs. per sq. ft.

Following the inquiry after the Tay Bridge disaster, the Board of Trade appointed a Committee to investigate the effect of wind pressure on railway bridges, and, as a result of the Committee's investigations, the B.O.T. recommended that in all designs of railway bridges allowance should be made for a maximum wind pressure of 56 lbs. per sq. ft. This figure, although seemingly high, readily justifies itself in cases where the structure is in an exposed situation, subjected to sudden gusts of wind, and more so when life and limb are dependent on the stability of the structure. In roof design, however, experiment has shown that it is hardly economical to make allowance for a greater maximum pressure than 40 lbs. per sq. ft., and that, only in very exposed situations. It is, however, only the component of this force normal to the roof surface which affects the stresses in the members composing the roof truss. In Continental practice it is usually assumed that the wind acts at an angle of 10°-15° to the horizontal, but in this country the direction of the wind is assumed horizontal. In determining the force exerted normally to the roof surface, the most usual method

is to make use of Hutton's formula, which is as follows:—

P = force of the wind, on a surface at right angles to its direction in lbs. per sq. ft.,

P_n = normal pressure in lbs. per sq. ft., on a surface inclined to the direction of the wind,

θ = angle of inclination of surface,

$$\text{Then } P_n = P(\sin.\ \theta)^{1\cdot84 \cos\theta - 1}.$$

When the wind blows horizontally, θ is equal to the slope or pitch of the roof.

Example.—If the wind, blowing horizontally, with a velocity corresponding to a pressure of 40 lbs. per sq. ft., impinges on a surface whose inclination to the horizontal is 35°, what will be the normal pressure in lbs. per sq. ft.?

$$P_n = P\ (\sin\ \theta)^{1\cdot84 \cos\theta - 1}.$$

$$\text{Sin } \theta = \sin 35^\circ = 0\cdot57 \text{ and } \cos\theta = \cos 35^\circ = \cdot82\,;$$

$$\therefore P_n = 40\ (0\cdot57)^{(1\cdot84 \times \cdot82) - 1},$$

$$= 40 \times (0\cdot57)^{\cdot5} = 40 \times \cdot756,$$

$$= 30 \text{ lbs. per sq. ft.}$$

It is not usual in this country to make any allowance for snow, on the assumption that the snow load is not likely to remain in the presence of the maximum wind load.

The following tables give the velocity of various winds and the normal pressures on variously inclined roofs for different horizontal pressures.

Description	Velocity in Miles/Hour	Description	Velocity in Miles/Hour
Calm . . .	3	Strong breeze .	34
Light air . .	8	Moderate gale .	40
Light breeze .	13	Fresh gale . .	48
Gentle breeze .	18	Strong gale . .	56
Moderate breeze.	23	Whole gale . .	65
Fresh breeze .	28	Storm . . .	75
		Hurricane . .	90

Angle of Roof	P=40	P=56	Angle of Roof	P=40	P=56
5°	5·0	7·3	50°	38·1	53·3
10°	9·7	13·5	60°	40·0	56·6
20°	18·1	25·6	70°	41·0	57·3
30°	26·4	37·1	80°	40·4	56·6
40°	33·3	46·7	90°	40·0	56·0

Determination of Stress due to Wind.—In determining the stresses in a structure due to combined dead and wind loads, we have the option of two methods of solution. We can either combine the dead and wind loads by finding their resultants, or we can treat each load separately, and then combine the two effects. By the former method we only get the maximum loads on the various members, while, by the latter method, we are enabled to get the variation in load on the various members, for which variation it may be necessary to make allowance, should it be considerable.

As the latter method is possibly the more easily understood, we will consider this method first and

then proceed to work out a few cases of combined dead and wind loads.

The first point to be considered is the determination of the total wind pressure acting on the truss, and this may be found thus—

Let L = length of rafter in ft.,
p = pitch of principals in ft.,
P_n = normal wind pressure in lbs. per sq. ft.,
W = total load due to wind, on any one truss in lbs.

$$\text{Then } W = L \times p \times P_n \text{ lbs.}$$

This load W is divided up among the joints along the rafter, and if N be the number of spaces, then the load on each end joint (*i.e.*, the apex and abutment) will be $\frac{W}{2N}$ lbs., while the load on each intermediate joint will be $\frac{W}{N}$ lbs.

In finding the reactions we have three distinct cases to deal with.

First Case.—Both ends of the roof truss are attached to the abutments by rigid fastenings, and consequently the nature of the reactions is quite indeterminate. It is, however, admissible to assume them to act parallel to the resultant wind pressure. We also assume that the resultant wind pressure acts through the centre of length of the rafter. Produce the line of action of W, and through the joints over the abutments draw two lines parallel to it. Set down *ab* parallel to AB and equal to it, to scale. Select a pole *o*, and join *ao* and *bo*. In the space A

draw a line *pq* parallel to *ao*, and in the space B a line *qv* parallel to *bo*. Draw the closing line *pv*, and from *o* draw a line *oc* parallel to *pv*. Then *ca* and *bc* give the magnitudes of the reactions R_1 and R_2 respectively.

Second Case.—In this type the wind is blowing from the right; the right-hand end of the truss is anchored to the abutment, while the left-hand end is mounted on a roller. This latter end is free to move, hence it can adjust itself until the reaction is vertical.

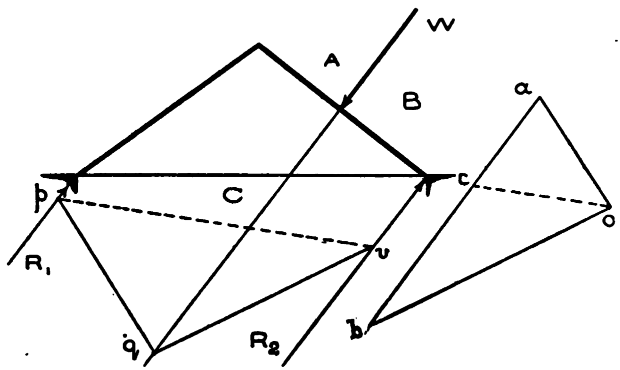


Fig. 88.

We saw in Chapter II. that if three forces, acting on a body, keep it in equilibrium, then these three forces must act through a common point.

The frame shown in Fig. 89 is in equilibrium under the action of three forces, viz., W, R_1, R_2. We know the directions of R_1 and W, R_1 acting vertically and W acting normally to the rafter through its mid-point, but all we know of R_2 is that its line of action passes through P.

Produce W and R_1 to intersect in O, and to satisfy the conditions of equilibrium stated above, the line of action of R_2 must pass through O as well as P, and hence line of action of R_2 is determined.

We have now determined three directions and one magnitude, hence it only remains to fix the magnitudes of R_1 and R_2. Draw *ab* parallel and equal to W, from *b* draw a line parallel to BC, and from *a* a line parallel to AC.

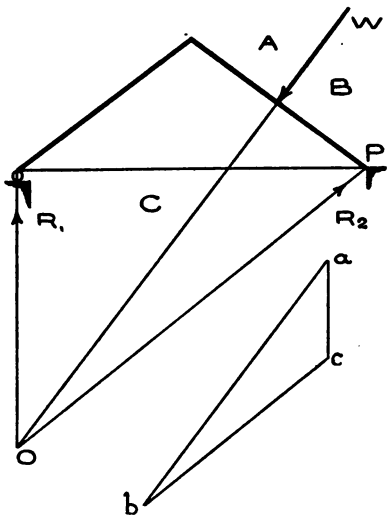


Fig. 89.

Then *ac* and *cb* give the magnitudes of R_1 and R_2 respectively.

Third Case. — In this type the wind is blowing from the left, while the fixings of the truss are as in the previous case. The method of construction and reasoning, as given in the previous case, apply equally well to this, and the student will have no difficulty with the solution if he studies carefully the constructions employed in Fig. 90.

In working out the stresses on the various members of a roof, due to a dead load and a wind load, the wind loads should be treated as acting first

from the right and then from the left, and the maximum values thus found should be used in the design.

Comparison of the Two Methods.—For the sake of comparison, we will now consider an actual case, solving the problem by the two methods, and comparing the results.

An engineering workshop is to be roofed with slates

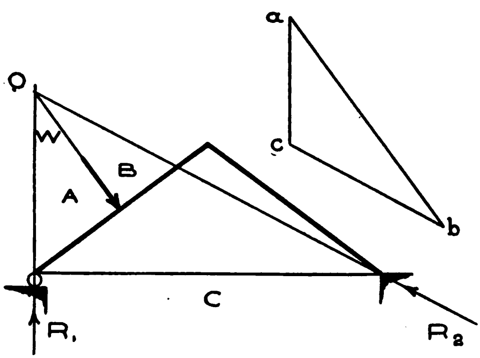


Fig. 90.

carried on wood boarding and purlins, the roof truss to be used being of steel and of the right-angled strut-type. The span of the building is 24 *ft. Pitch of the principals is* 8 *ft., and the pitch of the roof is* 30°. *The particulars of the loading are as under—*

> *Purlins,* 2 *lbs. per superficial ft.*
> *Boarding,* 3 *lbs. per superficial ft.*
> *Slates,* 8 *lbs. per superficial ft.*
> *Wind,* 40 *lbs.* (*horizontal*) *per sq. ft.*

Approximate weight of truss—

$$W_1 = \tfrac{3}{4}\, lP\left(1+\frac{l}{10}\right)$$
$$= \tfrac{3}{4} \times 24 \times 8\left(1 + \frac{24}{10}\right)$$
$$= 489{\cdot}6 \text{ lbs.} = 500 \text{ lbs. say.}$$

Length of rafter (from drawing) = 13·5 ft.

$\therefore$ Area of roof supported by each principal $= 2 \times 13{\cdot}5 \times 8$

$= 216$ sq. ft.

$\therefore$ Dead load equals—Purlins	$216 \times 2 =$	432
Boarding	$216 \times 3 =$	648
Slates	$216 \times 8 =$	1728
		2808

To allow for contingencies make dead load = 3000 lbs.

$\therefore$ Dead load, including truss = 3500 lbs. = W.

$\therefore$ Load at joint over abutment $= \dfrac{3500}{8}$

$= 440$ lbs.

$\therefore$ Load at intermediate joints $= \dfrac{3500}{4}$

$= 880$ lbs.

Wind load acting horizontally = 40 lbs. per sq. ft.

$$P_n = P\,(\sin\theta)^{1{\cdot}84\cos\theta - 1}\ (\theta = 30^\circ)$$
$$\log P_n = (1{\cdot}84\cos\theta - 1)\log\sin\theta + \log P$$
$$= \{(1{\cdot}84 \times {\cdot}86) - 1\}\log 0{\cdot}5 + \log 40$$
$$= ({\cdot}58 \times \bar{1}{\cdot}699) + 1{\cdot}602$$
$$= 1{\cdot}427$$
$$\therefore P_n = 26{\cdot}7 \text{ lbs. per sq. ft., normal to roof.}$$

$$\therefore \left.\begin{array}{l}\text{Normal wind load on} \\ \text{one side of truss}\end{array}\right\} = 8 \times 13{\cdot}5 \times 26{\cdot}7$$

$$= 2880 \text{ lbs.} = W_2$$

$$\therefore \left.\begin{array}{l}\text{Wind load at joint over} \\ \text{abutment and apex}\end{array}\right\} = \frac{2880}{4}$$

$$= 720 \text{ lbs.}$$

$$\therefore \left.\begin{array}{l}\text{Wind load at inter-} \\ \text{mediate joint}\end{array}\right\} = \frac{2880}{2}$$

$$= 1440 \text{ lbs.}$$

We can now set out these dead and wind loads at the joints and proceed to construct the stress diagram and so determine the maximum stress in the various members. We will assume, in this case, that the truss is fixed at both ends, and, in comparing the two cases, we will deal first with the combined dead and wind load. Referring to diagram No. 1, we first set down all the external loads on the truss on the load-line *ak*. Join the points *a* and *k*, and the line *ak* represents the resultant of all the external forces acting on the truss, its inclination giving the direction of the resultant and, hence also, the direction of the two reactions, R_1 and R_2. Before we can settle the magnitudes of R_1 and R_2, we must first find the point through which the resultant *ak* acts. Now we know that the dead load is uniformly distributed over the roof, and hence we can draw the resultant of the dead loads (*i.e.*, W) acting vertically through the apex of the roof. Again we know that the total wind load (*i.e.*, W_2) acts normally to one side of the roof and is uniformly distributed over that

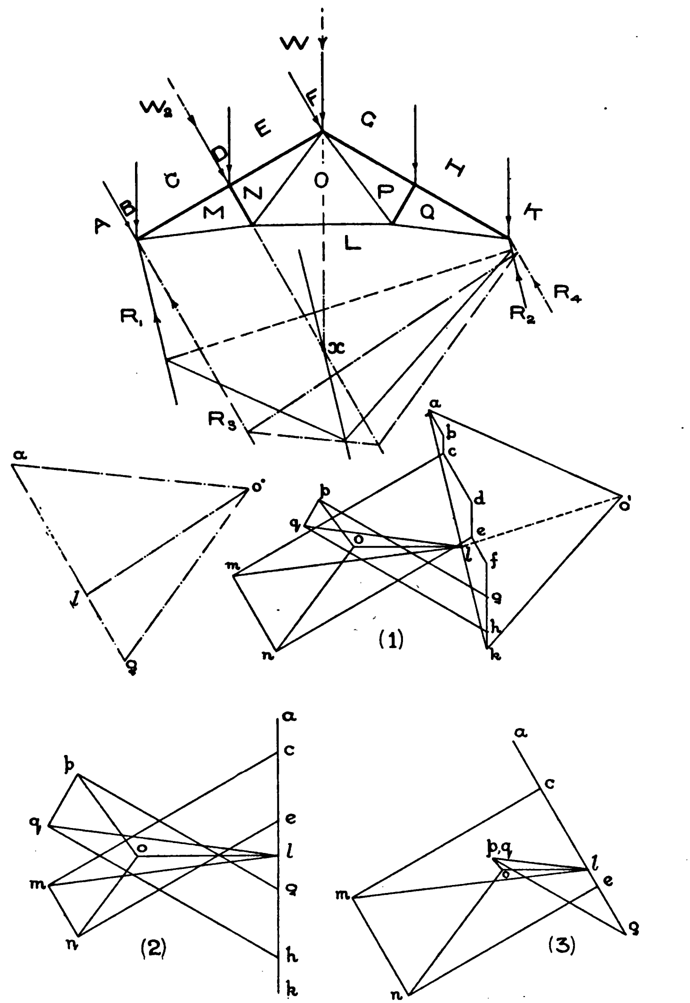


Fig. 91.

side, hence we can draw W_2, normal to the roof, through the mid-point of the rafter. The lines of action of these two loads will intersect in x, and x is therefore a point on the line of action of the resultant. We therefore draw through x a line parallel to *ak;* R_1 and R_2 can be assumed to act parallel to *ak*. Select a pole o', and join $o'a$ and $o'k$; draw the funicular polygon and complete it with the dotted closing line, parallel to which $o'l$ is now drawn; *kl* will equal R_2 and *la* will equal R_1.

The drawing of the stress diagram should now present no difficulty. The stresses should be scaled off and tabulated. In dealing with the dead and wind load separately, the same position diagram has been used to economize space. In making use of the lettering, the letter between the two forces has been omitted, thus, when dealing with the dead load, the load over the abutment becomes AC, the letter B being omitted along with the wind load AB. Similarly when dealing with wind loads we again omit B, and the wind load at the abutment is again AC.

Diagram No. 2 shows the stress diagram for dead loads only. The wind load diagram, as shown in diagram No. 3, calls for a little explanation. When the wind load is acting alone, we can assume it to be concentrated in a single force W_2 acting normal to the roof surface. The reactions (due to wind alone) R_3 and R_4 will therefore act parallel to this line, and it now remains to determine the

magnitudes of R_3 and R_4. Select a pole o'' and join $o''a$ and $o''g$. Draw the funicular polygon as shown chain-dotted, finally drawing $o''l$ parallel to the closing line; gl will equal R_4 and la will equal R_3.

Diagram No. 3 shows the stress diagram for the wind load, and it will be noticed that no stress is induced in the member PQ. Table I shows the stresses as scaled off from diagram No. 1, while Table II shows the stresses scaled off from diagrams Nos. 2 and 3. It will be seen that the figures, although agreeing fairly closely, still show some slight discrepancies. These latter might be accounted for by errors in drawing, but there is another cause which tends to bring about this difference. We made the assumption that the directions of R_1 and R_2 were each parallel to ka, but, by combining the reactions, as found from diagrams Nos. 2 and 3, we find that the resultant reactions, so found, do not coincide with R_1 and R_2 parallel to ka. We could quite easily find new values for R_1 and R_2 which would coincide with the values obtained by the combination of the reactions in diagrams Nos. 2 and 3. This would give a slightly different position of l, which would result in a slightly modified stress diagram. The indeterminate nature of the reactions, accentuated, to a large extent, by the yielding nature of the supporting walls and the twisting action of the fastenings, makes the exact solution of this case impossible, and makes the method adopted, in diagram No. 1, as justifiable as any other.

TABLE I

Member	Compression	Tension
CM	6250	—
EN	5800	—
GP	5000	—
HQ	5400	—
LM	—	5900
MN	2250	—
NO	—	3350
OP	—	1500
QP	800	--
QL	—	4100
OL	—	2800

TABLE II

Member	Diagram 2		Diagram 3		Total	
	Compress.	Tension	Compress.	Tension	Compress.	Tension
CM	3400	—	2800	—	6200	—
EN	2950	—	2800	—	5750	—
GP	3000	—	1950	—	4950	—
HQ	3400	—	1950	—	5350	—
LM	—	2950	—	3030	—	5980
MN	780	—	1450	—	2230	—
NO	—	1300	—	2000	—	3300
OP	—	1300	—	200	—	1500
QP	780	—	—	—	780	—
QL	—	2950	—	1220	—	4170
OL	—	1800	—	1100	—	2900

Roof with Free-end.—In the roof shown in Fig. 92, the right-hand end of the truss is carried on a roller, while the left-hand end is anchored to the

abutment. In finding the reactions in this case, we again make use of the funicular polygon, although the method employed is slightly different. On the windward side of the roof we first find the resultant of the forces acting at each joint, and then set down all the external loads in the line *af*. Select

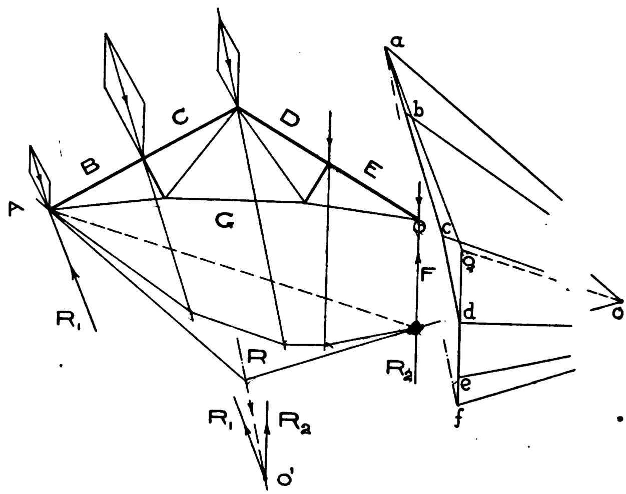


Fig. 92.

a pole *o*, and join *oa*, *ob*, *oc*, *od*, *oe* and *of*. Produce the lines of action of the various loads, and construct the funicular polygon as shown. A very important point arises here in connection with the construction of the funicular polygon. We know that the closing line must join the two reactions R_1 and R_2. Regarding R_2, we know that the line of action must be vertical,

while the only point known about R_1 is, that it must pass through the abutment joint under the load AB, its direction being still undetermined. Our closing line must terminate on the line of action of R_1, hence we are compelled to commence our funicular polygon from the abutment joint at the fixed end, and it should be noted that this rule always holds good. The reactions R_1 and R_2 are easily determined in the lines *ga* and *fg*.

Referring to the force diagram, it will be seen that the line joining *af* will represent the resultant (R) of all the external loads acting on the truss.

If we now produce the first and last lines of the funicular polygon (*i.e.*, the lines in the spaces A and F) and draw through their intersection a line parallel to *af*, then this line will give the line of action of the resultant (R). If the lines of action of R_1 and R_2 be produced to intersect in *o′*, it will be found that R passes through *o′*. The student can complete the diagram for himself by drawing the stress diagram for the various members.

Saw-tooth Roof.—Fig. 93 shows a saw-tooth roof subjected to wind and dead loads, the wind blowing from the left, while the right-hand side of the truss is fixed, the left-hand side being mounted on a roller. The student may experience some difficulty in constructing the funicular polygon in this case. The construction must be begun from the abutment joint under EF. Doubt may arise as to where the line parallel to *oc* should be drawn. It should be noted that *oc* is the *ray* between the force *bc* and *cd*, so that,

in the funicular polygon, the line parallel to *oc* will form the link connecting the lines of action of BC and CD. Note we get a crossed polygon in this case.

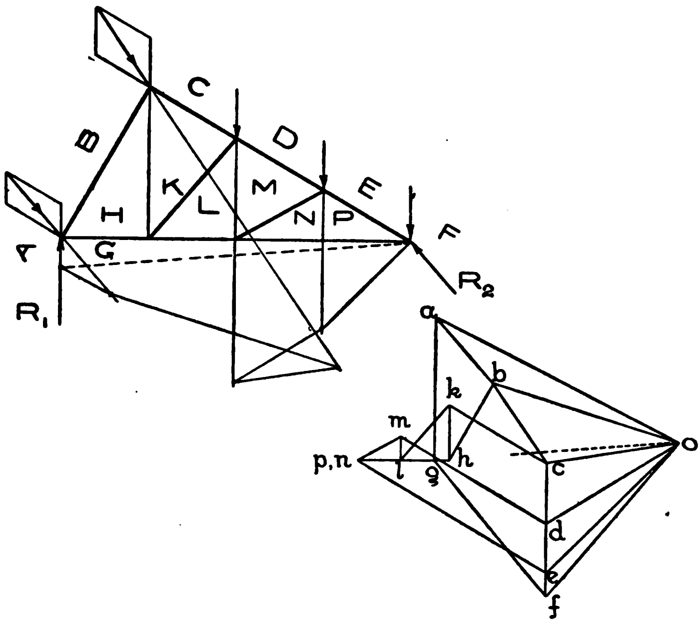


Fig. 93.

The finding of the reactions and the stresses is quite straightforward, and calls for no further explanation.

Island Station Roof.—The only important point in connection with this case is the method to be employed in finding the reactions. If we consider the dead loads alone, and assume these to be uniformly

distributed, then it is obvious that the two reactions will be equal, and each equal to half the total load. Consider now the effect which the resultant wind

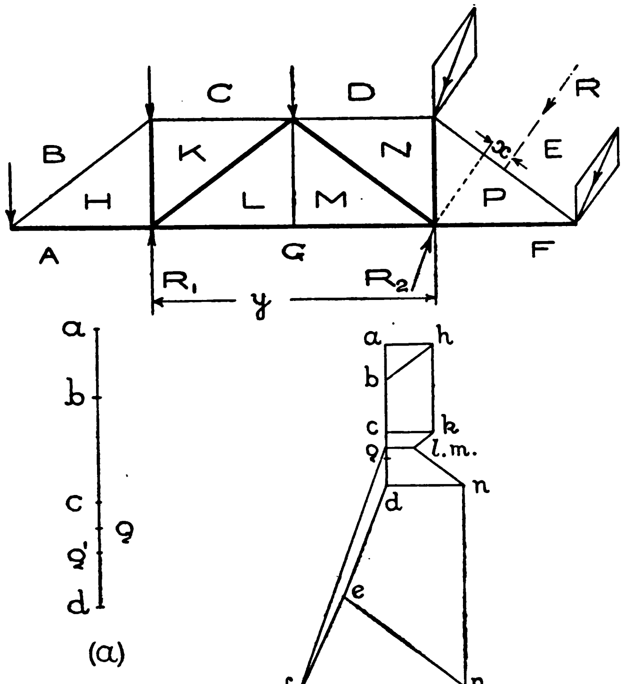


Fig. 94.

pressure (R) has on the truss; it will be seen that R has a moment about the right-hand support equal to $(R \times x)$. This would tend to cause a rotation about the right-hand support. This tendency to rotation

is counterbalanced by a downward force (say R_3) acting at the left-hand support, distant y from the right-hand support. Hence for equilibrium we must have—

$$R \times x = R_3 \times y.$$

$$\therefore\ R_3 = \frac{R \times x}{y}.$$

The dead-load reaction at the left-hand support is therefore diminished by R_3.

$$\therefore\ R_1 = \tfrac{1}{2} \text{ total dead load} - \frac{R \times x}{y}.$$

Coming now to the stress diagram, we now set down the external loads in the line *adf*. Fig. 94 (*a*) shows part of the load-line to an enlarged scale, and it will readily be seen that, if we were considering only dead loads, then $g'a$ would give the reaction R_1, where g' is the mid-point of the complete load-line (for dead loads only). Now let us suppose that the resultant wind pressure $(R) = 3600$ lbs.; it will be found from measurement that—

$$\frac{x}{y} = \frac{2}{24} = \frac{1}{12}, \text{ hence } R_3 = \frac{3600 \times 1}{12} = 300 \text{ lbs.}$$

We therefore mark off $g'g$ equal to 300 lbs., then ga equals the reaction R_1. The reaction R_2 is obtained by joining fg. The drawing of the stress diagram presents no further difficulties.

It must not be forgotten when working out stresses due to wind loads on roofs fixed at one end only, that when the wind blows from the right, very different

stresses will be obtained from those obtained when the wind blows from the left. For purposes of design, therefore, it is essential that the stresses be obtained for the wind blowing from each direction, and the maximum possible loads so obtained be used in the design calculations.

CHAPTER X

BRACED BEAMS AND GIRDERS

THE science of graphics finds another very useful application in the solution of problems involving the determination of the stresses in the various members of braced structures used in bridge work. The loadings to which such structures are subjected depend to a large extent on the purposes of the bridge, and to some extent on the locality. It is not intended to deal here with questions of loading, these being adequately discussed in works on Structural Design.

Braced Beam with Single Stanchion.— The simplest case with which we have to deal is the simple braced beam with single stanchion. An examination of Fig. 95 will show clearly that a beam braced in this way is in reality an inverted king-rod truss, with this important difference, that the nature of the stresses in the various members is now completely reversed; tension is changed to compression, and compression to tension.

Such a beam may be loaded in two different ways: either with a single concentrated load directly over the stanchion, or with a load distributed uniformly over the whole length of the beam.

The solution in the first case does not call for any

special explanation, and will be readily understood from the diagram. When a beam, so braced, carries a uniformly distributed load, it is customary to assume that the case resolves itself into one in which we have a beam continuous over two spans. The central reaction will then be equal to $\frac{5}{8}$ W, and this gives the load acting on the central stanchion, while the reaction at each abutment will be equal to $\frac{3}{16}$ W. Set down *ab*, *bc* and *cd* to represent $\frac{3}{16}$ W, $\frac{5}{8}$ W and $\frac{3}{16}$ W respectively, and complete the stress diagram as shown. It will be seen that it is only the central load which affects the stress in the members, but it must not be forgotten that *bf* does not represent the total compressive stress in the member BF, since a further compressive stress will be induced by the bending action caused by the portion of the load supported between the abutment and the central stanchion.

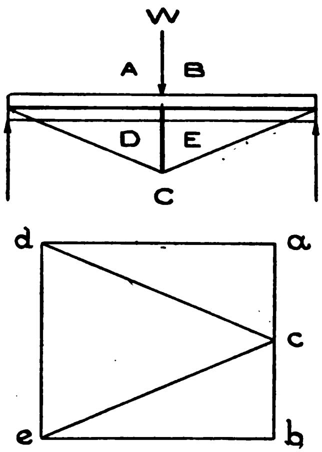


Fig. 95.

Braced Beam with Two Stanchions.—For longer spans, two stanchions, as shown in Fig. 97, are used, and the student will readily distinguish in this arrangement the inverted queen-post truss. This

type introduces one or two points of interest. If we assume pin joints at the head of the stanchions, it will at once be seen that the central panel is a deficient frame, and in order to make it complete, an additional member (shown dotted) will be required. If, however, the loads over the stanchions are equal and steady, it will be seen, from the stress diagram, that no stress is induced in FG, and hence, for this particular case, the additional diagonal member is not a necessity. By drawing out the stress diagram for unequal loading it will be seen that the diagonal then becomes necessary. For a variable load both diagonals must be added.

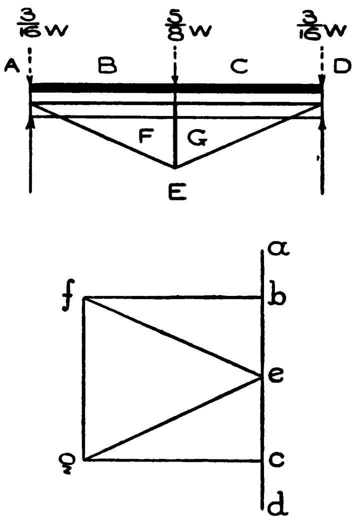


Fig. 96.

Trapezoidal Truss.—The trapezoidal truss, shown in Fig. 98, is a further modification of this type of beam, as used for still longer spans. The same remark regarding the central panel applies in this case, and for variable loading double counterbracing must be added. Some difficulty will be experienced by the student in solving this case, but the difficulty will be

easily overcome if it is remembered that the stress in any stanchion is equal to the load acting at its top end. Begin the stress diagram with the point marked x.

Fink Truss.—Another important modification of the bracing is that adopted in the Fink truss (Fig. 99), a type which is largely used in America. Taking the simplest case, in which we have three equal loads, AB, BC and CD, acting at the top ends of the stanchions, we first set down our load-line as *ab*, *bc*, *cd*. In order to obtain the stress diagram, we must begin with the point marked x, remembering, at the same time, that the stress in FK is equal to the load AB. This will give the triangle of forces *efk*. We can

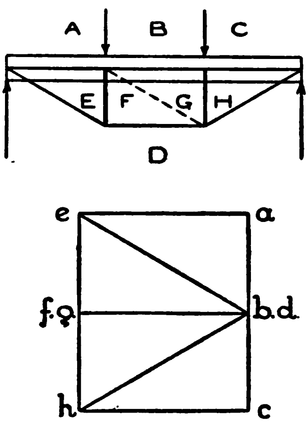


Fig. 97.

now solve the joint over the abutment, as given by the polygon *eagf*, and then pass on the joint under the load AB. The next point to be solved is that marked y, and this gives the polygon *ekhl*, thereby fixing the point l. The remainder of the solution is simple, and will be readily followed from the diagram.

The student is advised to work out a similar truss

in which the loads are unequal, say AB, BC, and CD, in the ratio 2, 3, and 4.

Bollman Truss.—The determination of the stresses in the members of the Bollman truss, shown in Fig. 100, will doubtless present some difficulties to the student, but the stress diagram can readily be

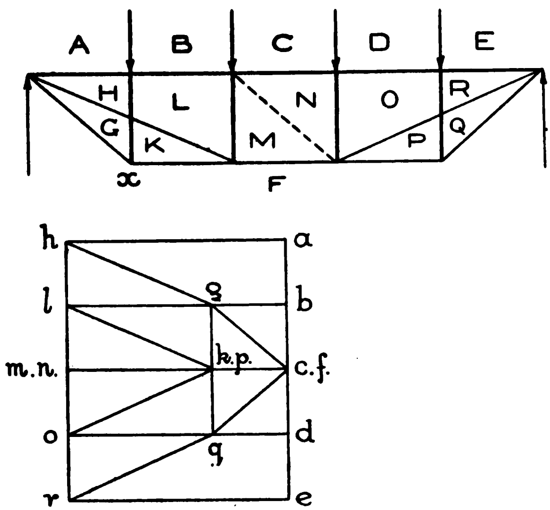


Fig. 98.

obtained by the following construction. Having set down the loads in the line *ad*, start with the joint marked *x*. From *e*, the reaction point, draw *ef* parallel to EF and *ek* parallel to EK. Place the vertical line *fk* equal to the load AB, with its ends *f* and *k* on the lines *ef* and *ek* respectively. Pass on then to the joint marked *y*, the solution of which is

given by the triangle *eno*. The joint marked *z* can next be solved in the polygon *kfgl*, remembering that the stress in EN equals the stress in KL. We can now pass on to the joint over the abutment, and thence to the joint under the load AB. The subse-

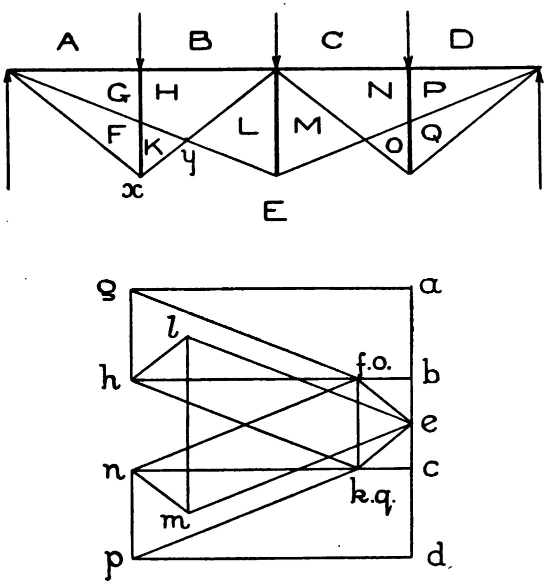


Fig. 99.

quent procedure will be readily followed from an examination of the completed force diagram. The student will find a valuable and instructive exercise in solving the case in which the loads AB, BC, and CD are unequal.

Warren Girder.—Of the various types of girders,

which are composed of top and bottom booms with web bracing, probably the simplest and best known is that invented by Captain Warren, and called, after him, the Warren girder. The simplest form consists of parallel top and bottom booms, with a single triangulation system. These triangles are usually

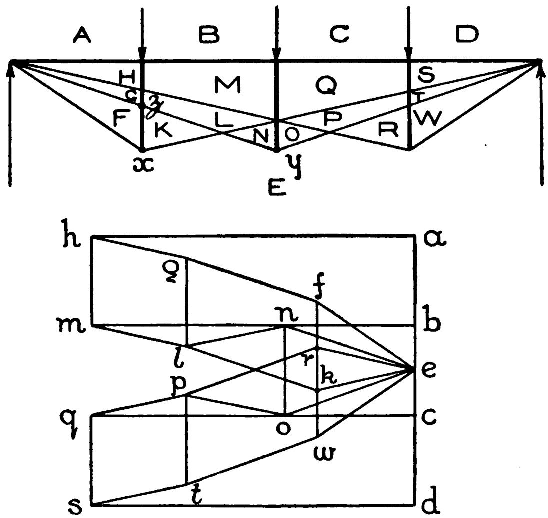


Fig. 100.

equilateral, although this particular shape is not a special feature of the girder; but one important point to note is, that in the Warren girder struts and ties are all of equal length. There are various ways in which a girder of this type may be arranged and loaded, but it will be sufficient for our purpose to consider only three cases.

Case I. Single Concentrated Load.—Fig. 101 shows a Warren girder of 5 bays, carrying a load W on the second joint from the top left-hand end.

Set down *ab* to represent W, and then fix the position of *c* from a consideration of the position of

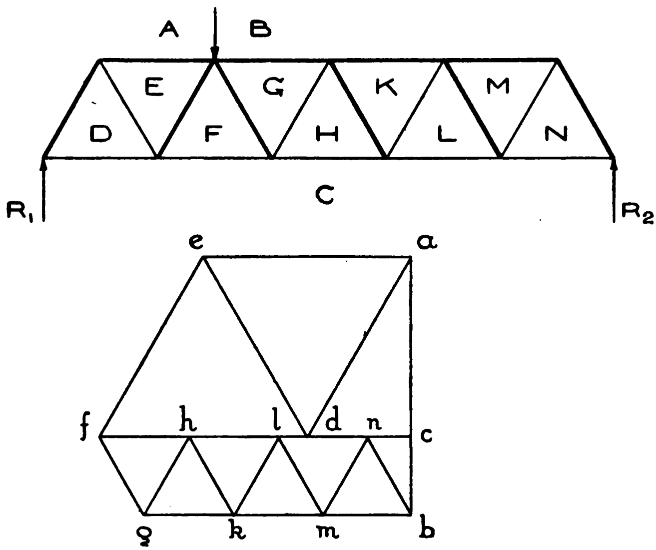


Fig. 101.

W. It will readily be seen that the line of action of W produced will cut the lower boom in the ratio 1·5 to 3·5, hence, by taking moments, we can find the value of the reactions, thus—

$$R_1 \times 5 = W \times 3{\cdot}5. \qquad \therefore\ R_1 = 0{\cdot}7W.$$
$$R_2 \times 5 = W \times 1{\cdot}5. \qquad \therefore\ R_2 = 0{\cdot}3W.$$

The point c must therefore be taken so that $ca = 0{\cdot}7W$ and $bc = 0{\cdot}3W$.

The solution of this problem should now present no difficulty. Start at the lower joint over the reaction

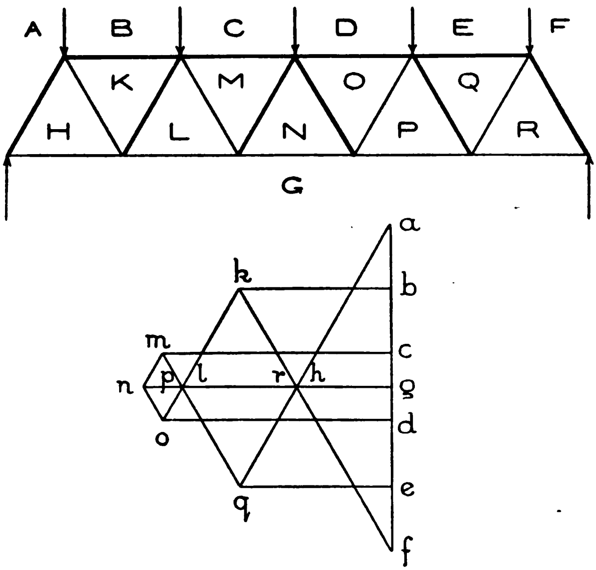


Fig. 102.

R_1, and then zigzag along the bracing, solving the various points as they are met.

Case II. Uniformly Distributed Load.—This case, as shown in Fig. 102, is simply an expansion of the previous case, every joint in the top boom now carrying a load, each load being of equal magnitude. The

reactions, in this case, will be each equal to half the total load.

Case III. Unsymmetrical Loading.—This is by far the most important case, involving, as it does, the use of the funicular polygon for the determination of

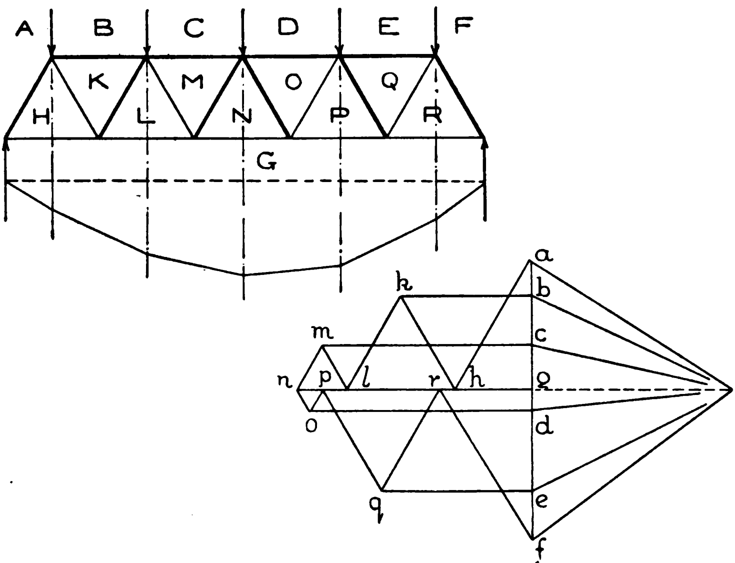


Fig. 103.

the reactions. The whole solution, however, both the determination of the reactions and the determination of the stresses, is perfectly straightforward, and the student should be able to solve this case without the aid of any further explanation.

Linville Girder.—It is a well-known fact that

while the strength of a tie-rod is not influenced by its length, the strength of a strut of a given section increases as the length is decreased. In arranging the members of a braced structure it will therefore be economical to arrange that the members subjected to compressive loads be made as short as possible. In furtherance of this idea, the Linville truss was introduced. It will be seen from Fig. 104 that the struts in this truss are vertical, thus giving the shortest possible length for any given depth of girder. In determining the stress, we first set down the loads in the line *af* and take the mid-point *g*, since the reactions are equal. We can then start with the joint at the left-hand abutment, and zigzag along the bracing, solving the joints as they come. This method presents no difficulties until the joint at the foot of the vertical NO is reached. The difficulty can, however, be overcome by assuming that the stress in the member MN is equal to the stress in the member OP. We can, of course, fix the point O if we remember that the stress in NO must be equal to CD; but, by treating the joint under the load CD first, the point O is fixed without any assumptions which might not be quite clear to the student. The remainder of the solution is quite straightforward, although, when the loading is symmetrical, it is unnecessary to draw more than one-half of the stress diagram.

Pratt Truss.—Fig. 105 shows a modification of this truss, known as the Pratt truss.

Lattice Girders.—If we take two simple Warren

girders, invert one of them, and superpose it on the other, we get an arrangement, as shown in Fig. 106, which illustrates one type of lattice girder. The fact that this type of girder is made up of two simple girders suggests at once a method of treatment, namely, to split the girder up into its components,

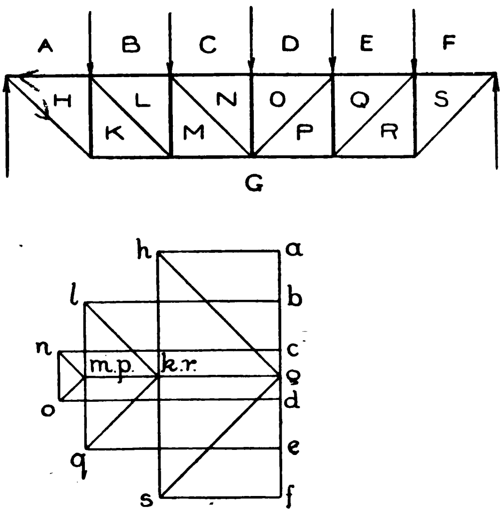


Fig. 104.

and solve each system separately, finally adding the stresses together. This method, although more liable to error, certainly has many advantages, but, in the majority of cases, time can be saved and errors obviated by drawing one stress diagram to serve both systems. The lattice girder shown in Fig. 106 is subjected to a system of symmetrical loads, and hence

M

the reactions will be each equal to half the total load. We can therefore set down the load-line and select *k*, the mid-point of its length. (In the diagram only part of the load-line has been drawn to economize space.) If we now attempt to draw the stress diagram, we are at once beset with the difficulty that every joint in the structure has acting on it a system of forces of which more than two facts are unknown. Hence it would seem that the case is insoluble. If

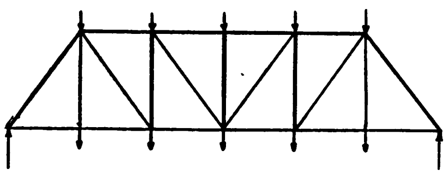


Fig. 105.

we examine the inverted triangulation system of the girder, it will be seen that this component supports the loads AB, CD, EF and GH, and that the supporting forces are transmitted, from the abutments, through the members AL and Hα. Therefore we can say—

$$\text{Stress in AL} = \text{AB} + \frac{\text{CD}+\text{EF}}{2}, \text{ and}$$

$$\text{Stress in H}\alpha = \text{GH} + \frac{\text{CD}+\text{EF}}{2}.$$

At the left-hand abutment we have a total reaction KA, which is produced by two distinct loads, namely, a downward load due to the upright system, trans-

mitted direct to the abutment, and a downward load due to the inverted system, transmitted to the abutment through the member AL. We can therefore calculate the stress in AL, and set it down as *al*

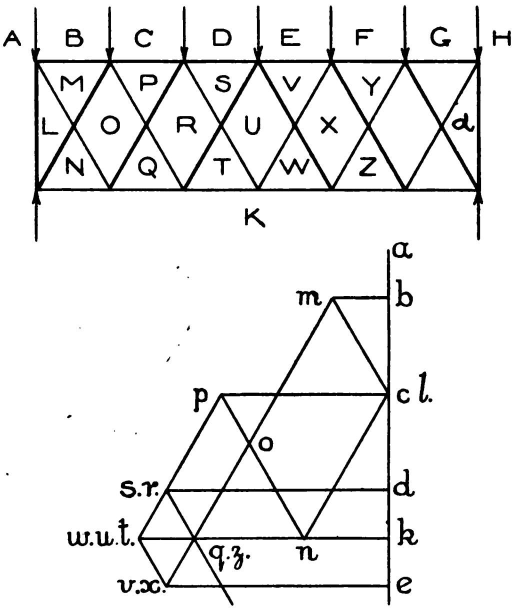


Fig. 106.

in the load-line; the point *l* coincides with *c* in this case. No further difficulty will be met, if the joints be treated in the following order: First go round the point at the left-hand abutment, then pass up the vertical to the joint under AB. Next pass along the sloping member LM, solving first the joint at the

intersection, and then the joint at the apex of the inverted triangle. From this point rise vertically to the joint under BC, and so on, tracing out a series of N-shaped paths from one end of the girder to the other. Only one-half of the stress diagram need be drawn.

The more general case of unsymmetrical loading is shown in Fig. 107. In this case the load AB is carried by the inverted system, while BC is carried by the upright system. The load AB, therefore, is the only one which affects the stress in AE, which can be calculated as follows. It will be noticed that the joint under AB divides the girder into two portions in the ratio 2 and 4, hence—

$$AE \times 6 = AB \times 4, \text{ or } AE = \tfrac{2}{3}AB.$$

The force AE is then set down in the line *ae*.

If any doubt should exist as to which of the loads affect the end verticals, the difficulty can be readily overcome by tracing out the diagonals forming the sides of the triangles. If the last diagonal finishes up directly on the abutment, then the load acting at the joint, from which the tracing began, has no effect on the end vertical, but if the last diagonal finishes up at the top end of the end vertical, then its effect must be calculated by moments as already explained. This, of course, refers to loads on both top and bottom booms.

The above rule does not, however, hold good in the case shown in Fig. 107 (*a*), where a series of vertical members have been introduced immediately below the loads. These verticals are intended to fulfil a

very important purpose, but whether the practical results attained are in accordance with the assumptions made, is a question which is open to considerable doubt. If we refer to Fig. 106, it will be seen that the stresses in any two intersecting diagonals are

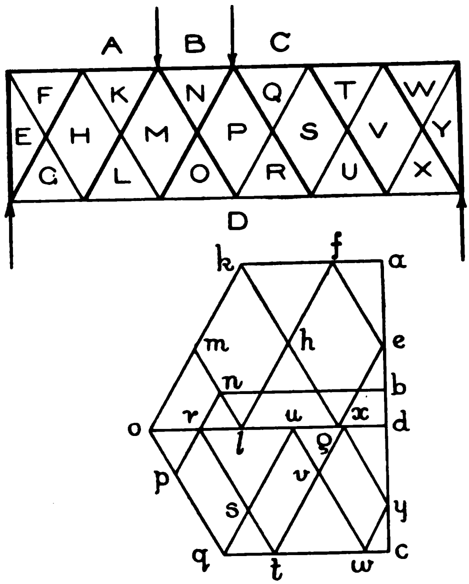


Fig. 107.

unequal, and the object to be attained by the introduction of the vertical members is the equalization of these stresses. The assumption is made that the vertical member carries one-half of the load acting at its top end. Thus, if we consider the load CD, it will

be seen that, without the vertical member, the whole of CD is carried by the inverted system; while, when the vertical member is added, the inverted system only carries $\frac{CD}{2}$, the other half being transmitted along the vertical, and finally carried by the upright system. When we come to consider all the loads and their effect on AL, we could imagine the whole of the bracing to be removed and the girder to consist of rigid top and bottom booms separated by the end verticals. One half of each of the loads BC, CD, DE, EF and FG would now be transferred to the corresponding points on the lower boom. The load on the lower boom is carried direct by the abutments, while the load on the upper boom is transmitted to the abutments by the end verticals. The load on AL is therefore equal to—

$$AB+\tfrac{1}{2}\left(\frac{BC}{2}+\frac{CD}{2}+\frac{DE}{2}+\frac{EF}{2}+\frac{FG}{2}\right),$$

and this is now set down in the line *al*.

The same procedure as was followed in the previous case is again adopted in drawing the stress diagram, it being noted that the stress in any vertical is equal to half the load at its top end. Thus, in the stress diagram, we draw *op* vertical and mark off *op* equal to $\frac{BC}{2}$.

Only a part of the stress diagram is shown, but it is sufficient to show that the stresses in the intersecting diagonals are now equal.

The effect of the verticals is therefore to equalize

the stresses in the diagonals, but it is very doubtful whether this result is attained in practice or not.

Cantilever Pier.—Fig. 108 shows a type of braced

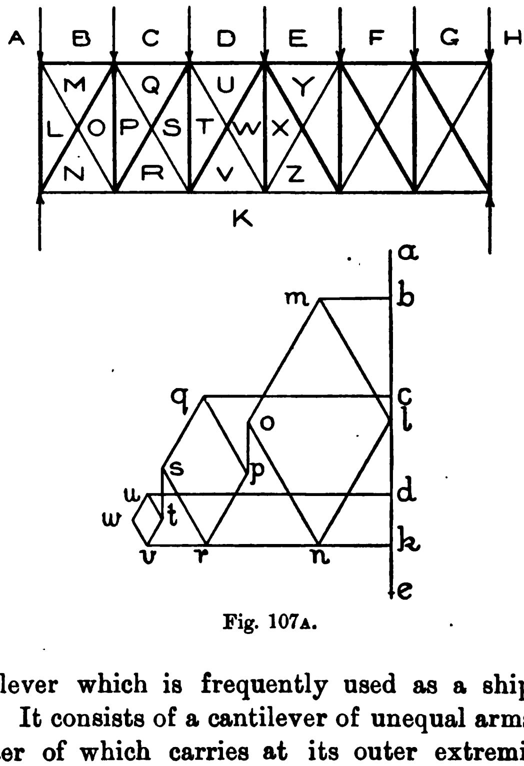


Fig. 107A.

cantilever which is frequently used as a shipping pier. It consists of a cantilever of unequal arms, the shorter of which carries at its outer extremity a balance weight (W), whose magnitude is such that the whole structure balances about a support which is immediately below the member MN. In solving

this particular case we have first to find the magnitude of the back balance weight, and, in doing so, we make use of the funicular polygon. The student will doubtless experience a little difficulty in setting down the loads, since, at the joint under MN, we have two forces acting, namely, an upward force or reaction, EF, which is equal to the sum of all the downward loads, back balance included, and a downward force, FG, equal to the load at the joint. In setting out the load-line for the construction of the funicular polygon, we delete the force EF and, with it, the letter F, hence the downward force at the joint becomes EG, and, as such, it appears in the load-line *ah*. Select a pole *o*; draw the rays and construct the funicular polygon, the last link, parallel to *oh*, cutting the line of action of W in the point *x*. Produce the first line of the polygon to cut the member MN in the point *y*. Now the last line of the link polygon, when produced, must pass through the points *x* and *y*, as shown. From *o* draw *oa′* parallel to *xy*, and *ha′* gives the magnitude of the back balance which will cause the whole structure to balance about the foot of the member MN. The only point calling for explanation in the stress diagram is the setting out of the load-line. Replace the force, EF, which is of course equal to *a′a*. Set down *ab*, *bc*, *cd*, and *de*; the next force is EF of magnitude *a′a*, acting upwards, hence measure *ef* upwards equal to *a′a*, then measure downwards *fg* and *gh*, leaving *ha* the back balance weight acting downwards. The remainder of the solution is quite straightforward.

Examples.

1. A trussed beam carries a load of 2 tons at the

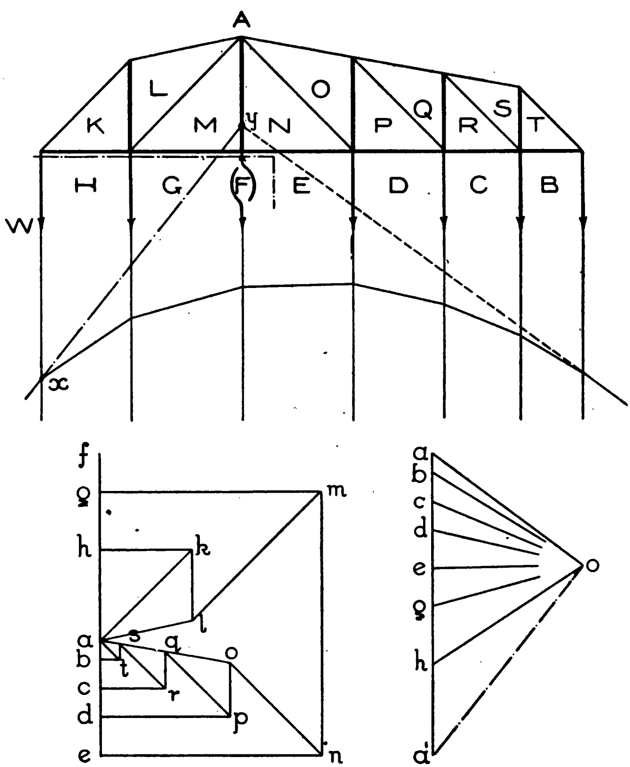


Fig. 108.

centre of an 18-ft. span. The beam has a single stanchion 2 ft. long directly under the load. Deter-

mine the stresses in the stanchion and ties, and the intensity of the compressive stress in the beam material if the section is 9 ins. × 6 ins.

2. A bridge, 60 ft. span and 10 ft. wide, is supported by two trusses of the type shown in Fig. 97. The stanchions, which are 8 ft. long, divide the truss into three equal divisions. The superstructure is attached to the trusses at the abutments and at the head of the stanchions, and the maximum load is equivalent to 180 lbs. per sq. ft. Find the stresses in the various members.

3. A trapezoidal truss has a span of 40 ft., and is divided into four equal divisions by stanchions 5 ft. long. The loads at the top of the stanchions, reading from the left, are 2, 3, and 2 tons respectively. Find the stresses in the members.

4. Assuming the loads in the previous case to be 2, 3 and 5 tons respectively, find the stresses in the members and determine which diagonal must be supplied to the central panel if the stress in the diagonal is to be tension.

5. A Fink truss, 30 ft. long, has four equal divisions, the stanchions being 4 ft. long. The load at the top of each stanchion is 2 tons. Find the stresses in the various members.

6. A railway bridge, crossing a river, has a span of 49 ft. The bridge is supported on two Fink trusses, each divided into 7 bays of 7 ft. each. The depth of the truss is 8 ft. It is estimated that the maximum dead load is equivalent to 35 tons, while the maximum live load is equivalent to $1\frac{1}{4}$ tons per ft. of length

Find the maximum stress in the members and the variation in stress.

7. Two Bollman trusses are used to carry a railway across a span of 70 ft. The trusses are divided into six equal panels and are 12 ft. deep. The superstructure weighs 140 tons, and the maximum live load which may come on the bridge consists of two trains, each weighing $1\frac{3}{8}$ tons per ft. of length. Determine the stresses in the various members of the trusses.

8. A Bollman truss, of 50 ft. span, has a depth of 12 ft. and is divided into five equal panels. Reading from the left-hand abutment, the loads are 2, 6, 4, 8, 10 and 4 tons respectively. Find the stresses in the various members.

9. A Warren girder, simply supported on a span of 40 ft., consists of four panels, the members of which are inclined at 60° to the horizontal. Reading from the left-hand support, the loads at the top joints are 4, 6, 5 and 4 tons. Find the stresses in the members.

10. If the girder, in the previous question, had, in addition to the loads on the top joints, a load of 2 tons on each of the lower joints, what would then be the magnitude of the stresses in the members?

11. A Linville truss has a span of 60 ft. The truss is 10 ft. deep and is divided into six equal panels. It carries a uniformly distributed load on the upper boom, equivalent to 2 tons at each of the abutments and 4 tons at each of the intermediate joints. Determine and tabulate the stresses in the members.

12. Determine the stresses in the above truss if, in

addition to the uniformly distributed load on the top boom, each of the joints in the lower boom carries a load of 3 tons.

13. A lattice girder, as shown in Fig. 106, has a span of 60 ft. Each triangulation system consists of three triangles, whose base angles are 45°. Find the stresses in the members if the loads at the joints above the abutments are each 2 tons, and at each of the intermediate joints, on the top boom, 4 tons.

14. Find also the stresses in the above girder if, in addition to the loads on the top boom, each joint in the lower boom carries a load of 3 tons.

15. A lattice girder has a span of 60 ft. and a depth of 16 ft., each triangulation system consisting of three triangles. The upper boom carries a load of 0·5 tons per ft. run, while the joints at the lower boom carry loads of 12, 18, 18, 18, and 12 tons respectively. Determine the stresses.

16. Draw out a cantilever pier, using the proportions of Fig. 108, and making the longer arm 80 ft. long. The diagonals all slope at 45°. Reading from the right-hand end, the loads are 2, 3, 4, 5, 6, 5 and W tons respectively. Determine the magnitude of W, and tabulate the stresses in the members, distinguishing ties from struts.

CHAPTER XI

CENTRE OF GRAVITY—NEUTRAL AXIS—RESISTANCE FIGURE—MODULUS OF SECTION—MOMENT OF INERTIA

Gravity.—It is a well-known fact that, if a body be allowed to drop from a height, it will move towards the earth with an ever-increasing velocity. Since motion is produced, it is evident that some force must be acting. The force by which a body is drawn towards the earth is spoken of as *the force of gravity.* To what extent can we apply our specification to this force? We know its direction—it acts vertically; we know its sense—it acts away from the body, drawing it towards the earth; further, if we hang the body on a spring balance, there will be registered, on the scale, the *weight* of the body which measures the magnitude of the force pulling the body towards the earth; hence the magnitude is known. What about the point of application? The point, at which the force of gravity is assumed to act, is termed the *centre of gravity* of the body. If it were possible to condense the material of the body into a single particle, then the C.G. can be looked upon as the position of that particle in its relation to the original body. A better term for this point is the *centre of mass*, but the former expression is

more common, and is invariably used in this connection.

We can, however, assume the body to be made up of an infinite number of particles, each one acted on by gravity, and the problem then resolves itself into the finding of the resultant of a system of like parallel forces.

The simplest method of finding the C.G. of a body is to suspend the body, so that it is free to take up any position under the action of gravity. Thus, in Fig. 109, we have an irregular - shaped plate, whose C.G. we desire to find. We would first suspend the plate at a point A, and, on the same pin, hang a plumb - bob. Transfer the vertical line, through A, to the plate. This line will contain the C.G. Select another point—say A^1—and repeat the process. The two verticals will intersect in a point O, which is the C.G. of the plate.

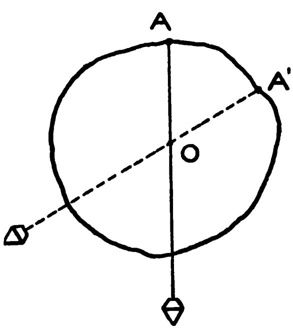


Fig. 109.

This experimental method is not always convenient, hence some other means must be adopted whereby we can find the point O. Before, however, considering the more complicated cases, we might, with advantage, study a few of the simpler geometrical figures. There are two simple cases of extreme importance, namely, the *triangle* and the *parallelogram*.

Triangle.—Fig. 110 shows a triangle ABC, whose C.G. it is desired to find. It is a matter of common knowledge that a wire of uniform cross-section will balance about its centre-point; its centre of length is its C.G. We can imagine the triangle, ABC, to be composed of a large number of fine wires placed side by side and lying parallel to the base BC, the lengths ranging from a maximum at the base to zero at the apex.

Bisect BC in D and join AD. The line AD will cut all the wires in their mid-points, hence every wire will be balanced on the line AD. The whole triangle, therefore, would balance if placed on a knife-edge coinciding with AD, and, consequently, the C.G. of the triangle must lie in AD. Similarly, draw BE to the mid-point of the side AC. The intersection O of the two medians gives the required C.G., DO being equal to $\frac{1}{3}$ AD.

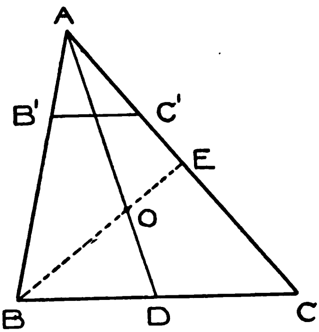


Fig. 110.

Rule.—*To find the C.G. of a triangle, draw any two medians and mark their point of intersection.*

Parallelogram.—Fig. 111 shows a parallelogram, ABCD, which, for our purpose, can be assumed to be composed of an infinite number of rhomboidal plates of equal size and weight. Bisect AB and DC and join EF, thus giving an equal number of plates on

each side of the line EF. Consider any two small plates, in the same horizontal row, equidistant from EF. Since these are of equal weight and equidistant from EF, their moments about EF will be equal and opposite, hence balance will be obtained. Every individual plate to the right of EF has a corresponding partner to the left of EF, hence all the plates on the one side will balance all the plates on the other. The C.G. of the parallelogram therefore lies on the line EF. In a similar way the C.G. must lie on the line GH. Hence the intersection O is the required C.G. It will be seen that the point O is also the intersection of the diagonals.

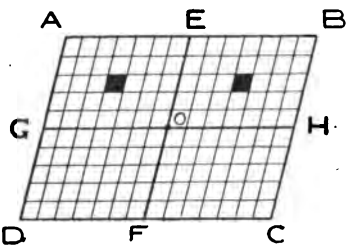


Fig. 111.

Rule.—*To find the C.G. of a parallelogram, draw the two diagonals and mark their intersection.*

Many simple geometrical figures can readily be subdivided into triangles and parallelograms, and the two preceding solutions readily suggest a means of solution. It will be sufficient, in this connection, to consider only one case. In Fig. 112 is shown a steel plate of the given shape, ABCD, and it is required to find the position of a hole, from which, if the plate be suspended by a cord, it will remain in a horizontal position. Draw BE parallel to AD, and AF parallel to BC. Find the C.G. of the triangle

BEC, and of the parallelogram ADEB, in the points P and Q respectively. Join QP. Find also the C.G. of AFD and ABCF in the points S and R respectively. Join SR. The intersection O is the position of the required hole, *i.e.*, the C.G.

Funicular Polygon Method.—If the shape of the figure be complicated, the above method soon leads to confusion. Complicated and irregular figures are

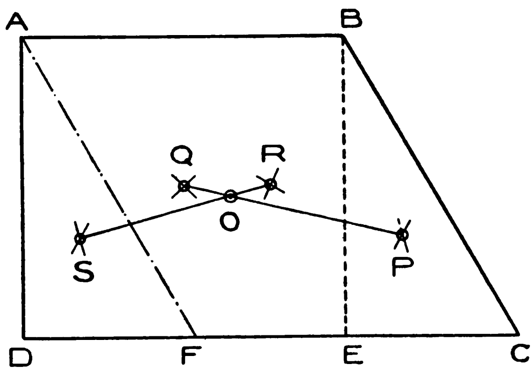


Fig. 112.

best solved by the aid of the funicular polygon. The problem simply resolves itself into finding the resultant of two systems of like parallel forces, the two systems being inclined to one another, preferably at right angles. Let us consider the case of the irregular-shaped plate shown in Fig. 113. Taking first the left-hand polygon, draw two vertical lines bounding the figure, and divide the horizontal distance between them into a number of equal parts, as

shown by the dotted vertical lines—in this case eight parts. Mark, on a horizontal line through the figure, the mid-points of these widths, and raise verticals. These verticals will be cut by the boundary curve of the figure. Measure these intercepts and set them down in the line *ak*, the lengths of the intercepts being increased or reduced in proportion as the case demands. Find the position of the resultant R_1 by the method already explained. Repeat the whole process in a direction inclined to the first, preferably at right angles, and so find R_2. The intersection of the two resultants, so found, will give the required C.G.

The above construction is applicable in all cases of plane figures of uniform weight. The case becomes simpler when the figure is symmetrical about a centre-line, as is the case in rail and beam sections. Only one funicular polygon is then required, for it is known that the C.G. must lie on the centre-line of the figure.

A section commonly used for cast-iron beam sections is shown in Fig. 114. In determining the C.G. of the section, we first divide the section up into three rectangular portions as shown, and find the C.G. of each portion. Through these points draw perpendiculars, and set down, as a load-line, *ad*, in which the subdivisions *ab*, *bc* and *cd* are each proportional to the area of each respective part of the section. The drawing of the funicular polygon is now quite straightforward, and the C.G. of the section is found in the point O, at which the resultant cuts the horizontal centre line.

When used as a beam section the arrangement is as shown in Fig. 115. The line N.A. through the C.G. of the section is called the *neutral axis.*

Neutral Axis.—*In all beam sections, so long as the stress does not exceed the elastic limit, the neutral axis always passes through the C.G. of the section.*

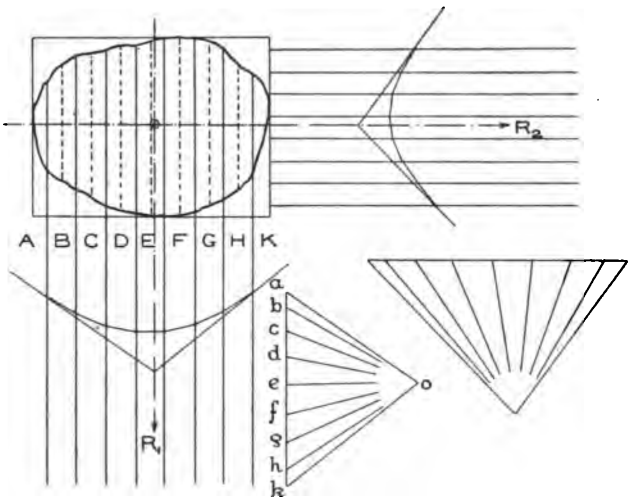


Fig. 113.

Fig. 116 shows a cast-iron channel section, and, as the metal is not symmetrically disposed about a centre line, it becomes necessary in finding the C.G. to draw the two polygons as shown. The student should work out this example for himself, assuming the section to have the following dimensions, namely: Compression flange, 4 in. × 1·5 in.; tension flange, 6 in.

$\times 2{\cdot}5$ in.; and web, 8 in. $\times$ 1·5 in. The position of the C.G. is 1·35 in. from the centre line of the web, and 1·0 in. from the centre of depth of the section.

Resistance Figures.—In dealing with the strength

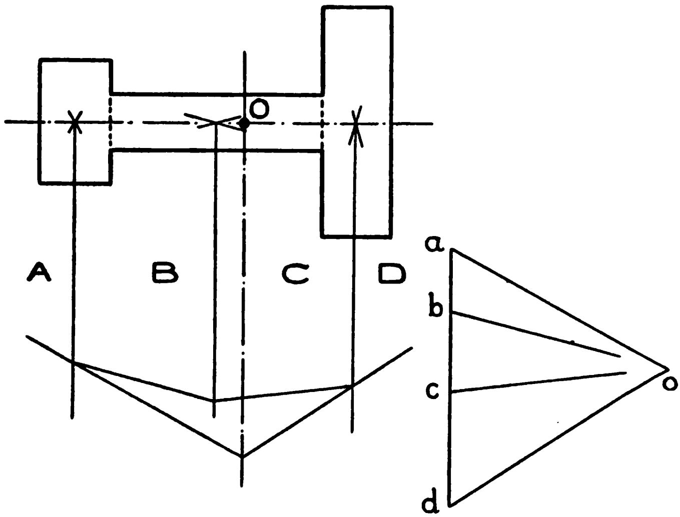


Fig. 114.

of beams subjected to bending moments, we have to deal with what is termed the *modulus of section* of the beam. Thus, let—

M = bending moment (lbs., ins.),

f = max. stress induced at sect. (lbs. p. sq. in.),

z = modulus of section (ins.^{3}).

$$\text{Then } M = f \times z.$$

The value of z varies with the shape of the section,

and, in determining its value for a given section, we can make use of certain simple graphical constructions. Consider the section, ABCD, as shown in Fig. 117, and let us assume that the maximum stress induced in a layer, at a distance y from the N.A, is f lbs. per sq. in. At any distance y' the stress induced will be less and will be equal to $f \times \frac{y'}{y}$. This is the stress intensity which is acting on the cross-sectioned strip,

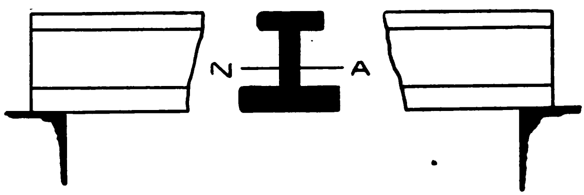


Fig. 115.

whose area we can take as a square inches. Now, obviously, if we reduce this area a, we will increase the stress per sq. in., since the total stress $\left(f \times \frac{y'}{y} \times a\right)$ will now be distributed over a smaller area. Keeping the depth of the strip the same, let us reduce its width until the area is a' and the stress intensity f lbs. per sq. in., then we have—

$$f \times \frac{y'}{y} \times a = f \times a'.$$

$$\therefore \frac{a'}{a} = \frac{y'}{y}.$$

Therefore, to give uniformity of stress over the whole

area, the width of the strips must be made proportional to their distances from the N.A. In other words, the ends of all the strips will be bounded by the triangles formed when the diagonals of the section are drawn. We have now a triangular area, on which there exists

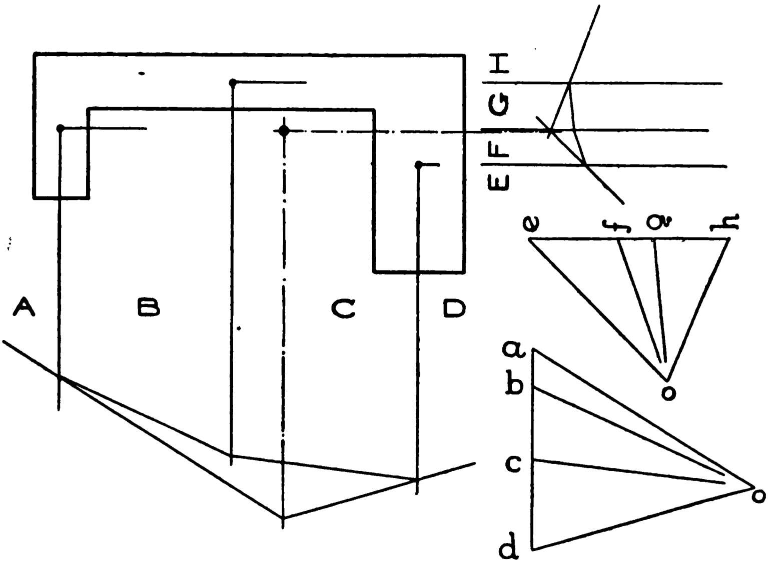


Fig. 116.

a uniform stress of f lbs. per sq. in., so that the *total stress* (P) is equal to the area of the triangle multiplied by the stress per sq. in.

$$\therefore\ P=\tfrac{1}{2}\left(b\times\frac{d}{2}\right)\times f=f\times\frac{bd}{4}.$$

This, of course, applies to each triangle above and below the N.A.

This force can be taken to act at the C.G. of each of the triangles, so that the arm $x = \frac{2}{3}d$. The moment of resistance of the section is therefore equal to—

$$f \times \frac{bd}{4} \times \frac{2d}{3} = f \times \frac{bd^2}{6} = f \times z.$$

The quantity $\frac{bd^2}{6}$ is the *modulus of section.* The figure composed of the two triangles, AOB and COD, is called the *resistance figure.* The finding of the modulus of section, therefore, resolves itself into two simple graphical operations, namely, drawing the resistance figure and then finding the C.Gs. of the areas above and below the N.A.

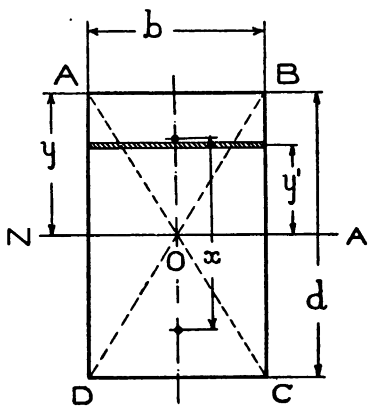


Fig. 117.

It will readily be understood that the metal in a beam section is being used to the best advantage when the stress at any interface is of uniform intensity over the whole area. Such an ideal condition is never attained in practice, and the resistance figure, in its relation to the section, shows at once how far the ideal condition has been realized. It will be seen that the rectangular section is not an economical one, since we have very little stress induced in the metal

near the N.A. Many sections are even worse, as will be seen from an examination of Figs. 118 and 119, which show the resistance figures for a square section with diagonal vertical and a round section respectively.

It might be well to explain briefly the general method of constructing a resistance figure for a given section. Referring to Fig. 120, the first thing to be done is to draw the N.A. This may be done by

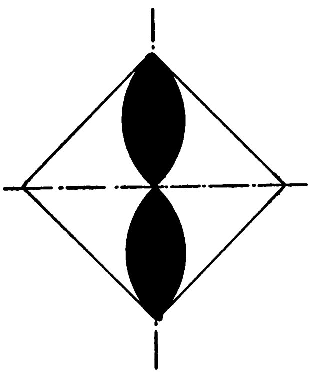


Fig. 118.

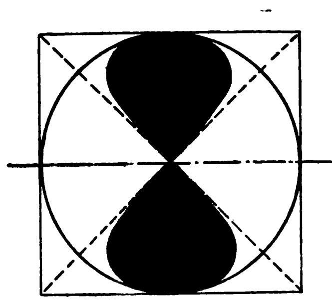


Fig. 119.

inspection in the given instance, but, for unsymmetrical sections, the funicular polygon must be drawn. Draw the lines AD and BC at right angles to the N.A., bounding the section on the left and right respectively. Complete the rectangle by drawing AB and DC parallel to the N.A. and equidistant from it, one of the lines, at least, being tangential to the section at the point most remote from the N.A. It should be noted that, in this case, as in all cases of

symmetrical sections, both lines will be tangential to the section. The wording, as given, is used in order that the construction may also be applicable in the case of sections which are unsymmetrical about the N.A. Draw any line, EF, parallel to the N.A., and cutting the section in E and F. From E and F raise verticals to cut AB in K and L respectively. Join KO and LO; these lines will cut EF in G and H, which are two points on the required resistance figure. Take a series of lines, such as EF, and so obtain sufficient points to enable the complete resistance figure to be drawn in. In general, it will be found most convenient to obtain the C.G. of each half of the resistance figure by the suspension method, and then transfer the positions to the drawing.

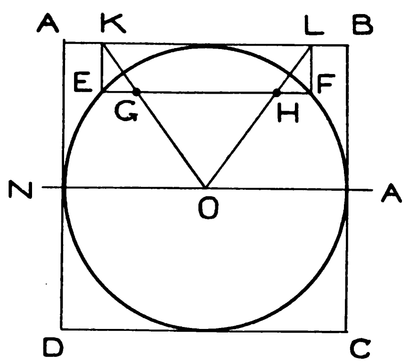


Fig. 120.

Let x = distance between C.Gs. in ins.

A = area, one-half of resistance fig. in sq. ins.

z = modulus of section in in.^{3} units

Then $z = A \times x$.

Figs. 121 and 122 show the resistance figures for a tee section and a cast-iron beam section respectively.

It must, however, be clearly borne in mind that,

although the resistance figure gives an indication of the efficiency of the disposition of the metal in the section, it does not necessarily follow that all parts of the section lying outside the resistance figure are useless.

If we examine Fig. 121, which shows the resistance figure for a tee section, we see at once that the metal in the web is more highly stressed than the metal in

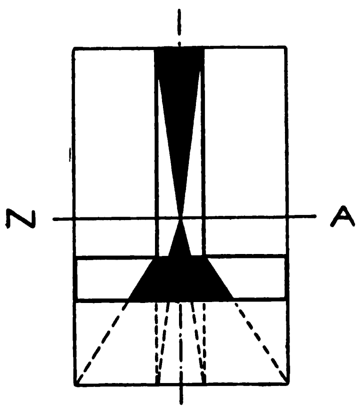


Fig. 121.

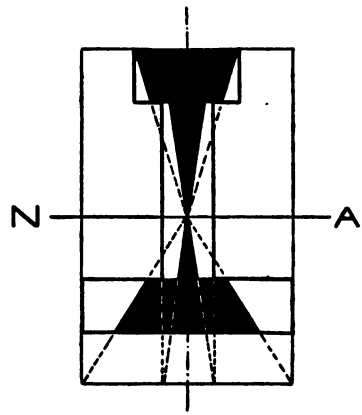


Fig. 122

flange, and, for the case of steel, which is, for all practical purposes, as strong in tension as compression, the tee section is obviously not an economical one to use as a beam. The case, however, is very different if the material of the section be cast-iron, whose strength in compression greatly exceeds that in tension. For cast-iron we have two resistance figures, one drawn on a compression stress base, as is shown

in Fig. 121, and another drawn on a tension stress base, as shown in Fig. 123. In the latter we again draw two lines, EF and GH, parallel to the N.A. and equidistant from it, but with this important difference, that one of the lines is now tangential to the section at a point *least* remote from the N.A.

To show the importance of the two different resistance figures, we might apply this particular section to a given case.

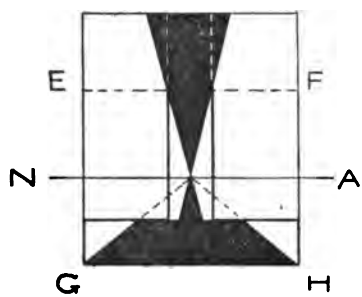


Fig. 123.

Example.—*A cast-iron tee section, 3½ in. deep and 3 in. wide, is ⅝ in. thick throughout. What safe load would this section carry at the centre of a 3-ft. span, assuming the web of the section placed uppermost?*

$f_t = 2000$ lbs. sq. ins. and $f_c = 6000$ lbs. sq. ins.

(*a*) *Compression stress base.*

Scaling from Fig. 121, when drawn full size, it was found that the triangular area above the N.A. $= \cdot 75$ sq. in.; distance between C.Gs. $= 2 \cdot 50$.

$$\therefore\ z_c = 2 \cdot 5 \times \cdot 75 = 1 \cdot 875$$

$$\frac{W\,L}{4} = f \times z_c.$$

$$\therefore\ W = \frac{f \times z_c \times 4}{L} = \frac{6000 \times 1 \cdot 875 \times 4}{12 \times 3}$$

$$= 1250 \text{ lbs.}$$

Therefore, as far as the compression stress is concerned, we could safely load the beam to 1250 lbs.

(*b*) *Tension stress base.*

Scaling from Fig. 123, when drawn full size, it was found that the triangular area above the N.A. $=1\cdot5$ sq. ins.

$$\therefore\ z_t=2\cdot5\times1\cdot5=3\cdot75,$$

$$\frac{WL}{4}=f\times z_t.$$

$$\therefore\ W=\frac{f\times z_t\times4}{L}=\frac{2000\times3\cdot75\times4}{12\times3}$$

$$=830\text{ lbs.}$$

Therefore, safe load is 830 lbs. at centre, this being the load which induces the maximum tension stress allowable.

It should be noted that a well-designed section in cast-iron will have the maximum compression and tension stresses acting simultaneously.

From an examination of Fig. 123, it will be seen that, at the outside layers of the web, the width of strip necessary to keep the stress down to a given limit is wider than the section at that point can provide, and it would therefore appear that the section will be overstressed at that point. It must be borne in mind, however, that the stress at this point is a compression, whose limiting value is very much higher than the limiting value in tension.

Fig. 124 shows the resistance figure for a rail section. The area of one-half of the resistance figure is

3·05 sq. ins., and the distance between the centres of the two C.Gs. is 4·5 ins.

$$\text{Modulus of section} = 3{\cdot}05 \times 4{\cdot}5 = 13{\cdot}8.$$

(*Note.*—Full-size depth of section = 6 ins.)

Moment of Inertia.—In determining the deflections produced in a beam on the application of external loads, we have to deal with what is termed the *moment of inertia* of the section. This quantity depends on the size and shape of the section, and must be calculated with reference to some fixed axis; in beam sections, the neutral axis is the axis of reference. Graphical methods can again be employed in determining this important quantity. If we have a body situated, with respect to an axis xy,

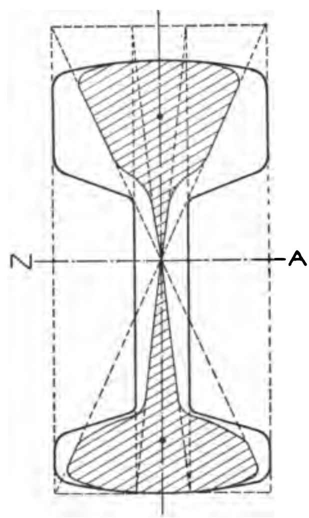


Fig. 124.

as shown in Fig. 125, then the moment of inertia of any little particle m at a distance r from xy will be equal to mr^2. The moment of inertia of the whole body with respect to $xy = I = \Sigma mr^2$. Referring to Fig. 126, let m be a particle of which we wish to find the moment of inertia about P, P being the end elevation of the axis

of reference. Set down *ab* to represent *m*, select a pole *o*, preferably making H an even number of ins. Join *oa* and *ob* and draw *mc* and *md* parallel to *oa* and *ob* respectively. Then, as explained in Chapter IV., we have $ab \times r = cd \times H$; $(ab \times r)$ is the *first moment* of *m* about P.

Now set down *cd* and select a pole *o′*, making, if

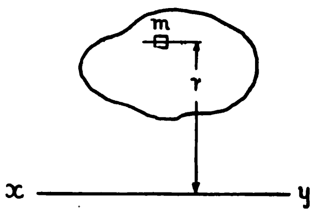


Fig. 125.

possible, H^1 equal to H. From *o″* draw *o″e* and *o″f* parallel to *o′c* and *o′d* respectively.

Then again we have $ef \times H^1 = cd \times r$.

$$\text{But } cd = \frac{ab \times r}{H}.$$

$$\therefore\ ef \times H^1 = \frac{ab \times r}{H} \times r.$$

$$\therefore\ ef \times H^1 \times H = ab \times r \times r$$
$$= ab \times r^2 = mr^2 = I_P.$$

The quantity $(ab \times r^2)$ is termed the *second moment* of *m* about P, or the moment of inertia of *m* about P. If the above diagrams are drawn so that 1 in. = 1 lb., and so that *r* is full size, then there will be no difficulty in finding the value of I_P, but, in general, both

quantities are drawn to scale, and some confusion may arise as to the true value of I_P.

Let the force scale be 1 in. $= p$ lbs.
Let the space scale be 1 in. $= q$ ft.

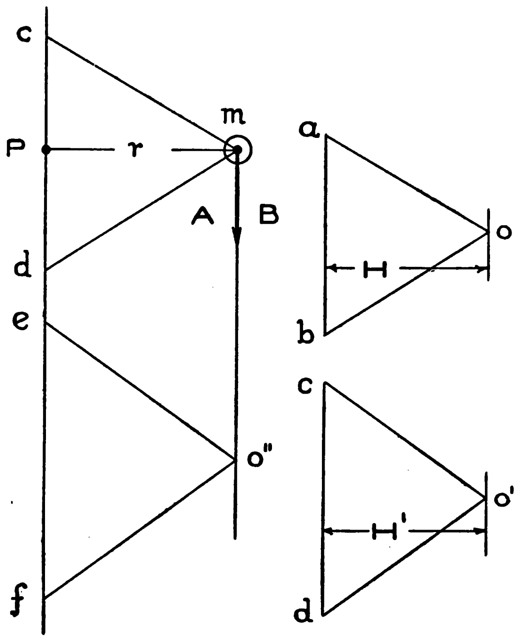


Fig. 126.

Then we have—

$$(m \times r) = cd \times 12q \times p \times \frac{H}{12},$$
$$= (cd \times q \times p \times H) \text{ lbs. ft.}$$
$$\therefore (m \times r) \times r = (cd \times q \times p \times H) \times r.$$

Now from the diagram it will be seen that—

$$\frac{cd}{ef}=\frac{\mathrm{H}^1}{\frac{r}{q}} \quad \because r\ (\text{ins.})=\frac{r\ (\text{ft.})}{q}.$$

$$\therefore cd = ef\times\frac{\mathrm{H}^1\times q}{r}.$$

$$\therefore mr^2=ef\times\frac{\mathrm{H}}{r}\times q^2\times p\times \mathrm{H}\times r,$$

$$=(ef\times q^2\times p)\times \mathrm{H}_1\times \mathrm{H} \text{ lbs. ft.}^2 \text{ units.}$$

Thus let m equal 15 lbs. and r equal 12 ft. The scales used in the original diagram are 1 in. equals 10 lbs., and 1 in. equals 8 ft., and $ef=2{\cdot}15$ ins.

$$\therefore \mathrm{I_P}=ef\times q^2\times p\times \mathrm{H}_1\times \mathrm{H},$$

$$=2{\cdot}15\times 8^2\times 10\times 1{\cdot}25\times 1{\cdot}25 (\mathrm{H}_1=\mathrm{H}=1{\cdot}25 \text{ in.}),$$

$$=2144 \text{ lbs. ft.}^2 \text{ units,}$$

$$mr^2=15\times 12\times 12,$$

$$=2160 \text{ lbs. ft.}^2 \text{ units.}$$

Moment of Inertia of a System of Forces.—The above construction can readily be applied to determine the moment of inertia of a system of forces, such as are shown in Fig. 127, but it will be seen that the moment of inertia, so found, does not equal the moment of inertia of the beam section (shown dotted) from which these individual forces have been derived. A close approximation can, however, be obtained, by dividing the section into a large number of strips, whose axes are parallel to the N.A. The section in this case is of cast-iron, the compression flange being 8 ins. × 4 ins. = 32 sq. ins., the tension flange 6 ins. × 2 ins. = 12 sq. ins., and the web 12 ins. × 2 ins. = 24 sq. ins.

We first set down ab, bc and cd equal to 32, 24 and 12 respectively to some scale.

Select a pole o, making H an even number of inches, and draw the funicular polygon. Draw in the resultant R, which will of course pass through the C.G. of the section. Let us assume that the line

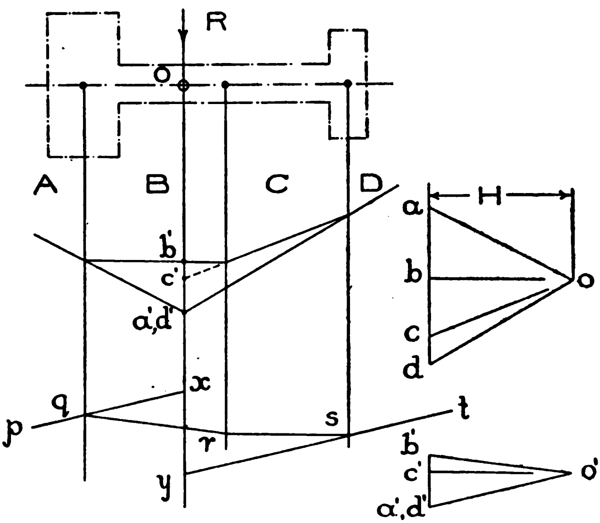


Fig. 127.

of action of R is, in this case, the axis of reference, about which we wish to find the moment of inertia of the system. If necessary, produce the links of the funicular polygon, so that each one cuts the axis of reference. Then we have—

Moment of AB about O $= +(a'b' \times \text{H})$,
Moment of BC about O $= -(b'c' \times \text{H})$,
Moment of CD about O $= -(c'd' \times \text{H})$.

O

These intercepts, multiplied by H, give the first moments of the respective forces about the axis of reference.

The intercepts are now set down and a fresh pole o' selected, and a second funicular polygon drawn as *pqrst*. The polar distance H^1 should, if possible, be made equal to H. Produce *pq* and *ts* to cut the axis of reference in x and y respectively. Then the required moment of inertia $= xy \times H \times H^1$, xy being measured to the proper scale.

The following dimensions were taken from the original drawing:—

Point O – 7·5 ins. from under side of section

$H = H^1 = 3$ ins.

Force scale 1 in. = 20 lbs.

Linear scale 1 in. = 2 ins.

$xy = 3{\cdot}1$ ins.

∴ Moment of inertia of given system of forces about O—

$$= 3{\cdot}1 \times 2^2 \times 20 \times 3 \times 3$$
$$= 2232 \text{ lbs. ins.}^2 \text{ units.}$$

The calculated I for the given beam section is 2532 ins.^{4} units.*

It will be seen that the discrepancy is considerable, so that obviously this particular solution cannot be applied to find the moment of inertia of a beam section.

Much greater accuracy can, however, be obtained by taking a larger number of sections, and proceeding on the same lines.

* Note the difference between *lbs. inches*^{2} and *inches*^{4} *units.* In the previous case we have taken forces as proportional to areas, *i.e.* to *inches*^{2}.

The moment of inertia of a section can be determined from a measurement of the area of a specially constructed funicular polygon, the method being due to Professor Mohr.

The construction is shown in Fig. 128. The section is first divided up into a number of parts as shown, and perpendiculars dropped from the C.G. of each portion. A load-line is set down, as shown, in the line *ay*. Let the total length of this line equal x ins., then the polar distance H is made equal to $\frac{x}{2}$ ins. Construct the funicular polygon and measure its area by means of a planimeter. Measure also the area of the section.

Let A_1=area of funicular polygon in sq. ins.,
A_2=area of section in sq. ins.,
I=moment of inertia in in.^{4} units.

$$\text{Then } I = A_1 \times A_2.$$

It should be further noted that $z=\frac{I}{y}$, where y= the distance of the greatest strained fibres from the N.A.

The section shown is the same as in Fig. 124, and the undernoted dimensions were taken from the full-size drawing.

Area of funicular polygon (A_1)=4·6 sq. ins.,
Area of rail section (A_2)=10·0 sq. ins.,
Distance of extreme fibres from N.A.=3·25 ins.

$$\therefore I_{N.A.} = 4{\cdot}6 \times 10 = 46 \text{ ins.}^4 \text{ units.}$$

$$\therefore z = \frac{46}{3{\cdot}25} = 14{\cdot}1.$$

It will be seen that this figure is practically the same as that obtained from the resistance figure shown in Fig. 124.

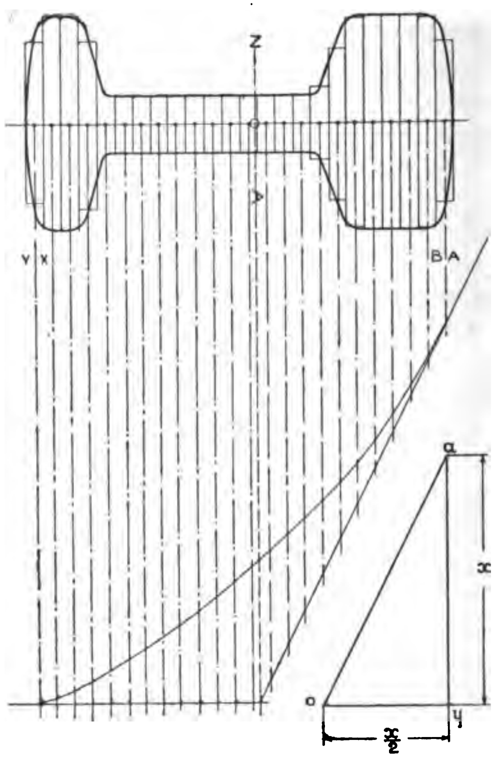


Fig. 128.

Ferro-concrete Beam Section.—Fig. 129 shows an interesting adaptation of Mohr's method to the case of a ferro-concrete beam section. In such a

beam the concrete is arranged to take the compressive stresses, while the steel reinforcement takes all the tension stresses. Experiment has shown that—

$$\frac{\text{Elastic modulus of steel}}{\text{Elastic modulus of concrete}}=15.$$

In determining the values of I and z for such a

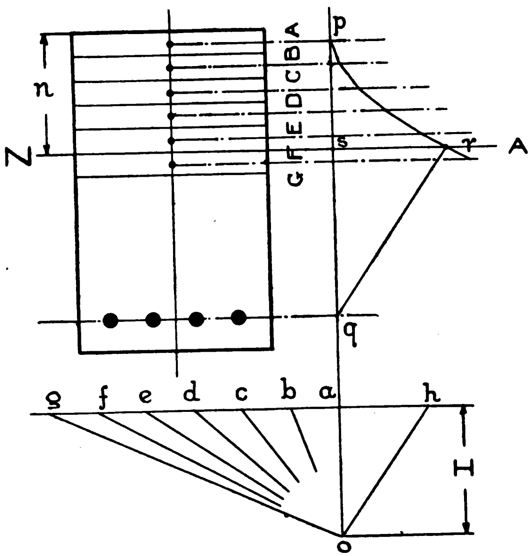


Fig. 129.

section, we first divide up the compressive side of the beam into a series of strips, and draw through the C.G. of each strip a horizontal line. Measure the area of each strip and set out the areas in the line ag; it should be noted that only a portion of the section need be divided into strips; the reason will be obvious from the

diagram. Through a raise a vertical and mark off $oa = \mathrm{H}$, making this preferably an even number of ins. Measure the area of the steel reinforcing bars.

Let m = number of steel bars,

a' = area of each bar in sq. ins.

Then total area $= (m \times a')$ sq. ins.

Set off $ah = (15 \times m \times a')$ sq. ins., being careful to set ah off to the same scale as was used for ag. Join the various points to the pole o, and construct the funicular polygon pqr as shown. The horizontal through r will give the N.A. of the section.

Then I (for section about N.A.) = area $pqr \times 2\mathrm{H}$.

In general, the section will be drawn to some scale, and this must be borne in mind when measuring the area pqr. The area pqr is measured to the scale to which the section has been drawn, and H is measured to the scale to which ab, bc, etc., have been set out.

Thus if the beam section be drawn half size, the area of the section and also of the funicular polygon will only be one quarter of its full-size value, and if ah be drawn to a reduced scale, then obviously the length sr will be reduced in the same proportion.

$$\text{Now I (for steel)} = (15 \times \text{steel area}) \times sq^2$$
$$= ah \times sq^2.$$

$$\text{But } \frac{sr}{sq} = \frac{ah}{ao} = \frac{ah}{\mathrm{H}}.$$

$$\therefore\ sq = \frac{sr \times \mathrm{H}}{ah} \quad \text{and} \quad ah = \frac{sr \times \mathrm{H}}{sq}$$

$$\therefore\ ah \times sq^2 = sq \times sr \times \mathrm{H},$$
$$= 2 \times \text{area } qsr \times \mathrm{H}.$$

$$\therefore\ \text{I (steel)} = \text{area } qsr \times 2\mathrm{H}.$$

Similarly I (total) = area $pqr \times 2H$.

Now, let k = number of reductions of section,

m = scale of load-line ag,

A_3 = actual area of pqr in sq. ins.

Then $I\text{ (total)} = (A_3 \times k^2 \times m) \times 2H$,

$= (A_3 \times k^2) \times (2H \times m)$ in.^{4} units.

The undernoted dimensions were taken from the original drawing of Fig. 129:—

Area of pqr $(A_3) = 0{\cdot}8$ sq. in.

Area of qsr $= 0{\cdot}54$ sq. in.

Scale of section, $1\frac{1}{2}$ ins. = 1 ft. $\therefore\ k = 8$.

Scale of load line 1 in. = 48 sq. ins. $\therefore\ m = 48$.

Polar distance $H = 1$ in.

Area of steel $\times 15 = 2{\cdot}2 \times 15 = 33$.

Distance $sq = 10{\cdot}1$ ins. (full size).

$$\therefore\ I\text{ (total)} = (0{\cdot}8 \times 8^2) \times (2 \times 1 \times 48),$$
$$= 0{\cdot}8 \times 64 \times 96,$$
$$= 4915 \text{ in.}^4 \text{ units.}$$

Again, checking for steel only—

$$I\text{ (steel)} = \text{area } srq \times k^2 \times 2H \times m$$
$$= 0{\cdot}54 \times 64 \times 2 \times 48 = 3320 \text{ in.}^4 \text{ units.}$$
$$\text{and } mr^2 = (2{\cdot}2 \times 15) \times 10{\cdot}1^2$$
$$= 33 \times 102$$
$$= 3366 \text{ in.}^4 \text{ units.}$$

The slight difference can be accounted for by errors in drawing, especially when the scale of the original drawing is one-eighth full size.

It is now a simple matter to determine the modulus of section.

Let f_c = crushing strength of concrete in lbs. p. sq. in.
n = distance of greatest strained particles from N.A. in ins.

Then $z = \dfrac{\text{I (total)}}{n}$, and the mom. of resist. $= \dfrac{f_c \times \text{I}}{n}$

Examples.

1. Draw any irregular figure, such as is shown in Fig. 113. Find, by the funicular polygon method, the C.G. of the area.

Carefully cut out the figure and check your result by the suspension method.

2. A steel plate is composed of two circular discs, joined together by a rectangular piece, whose centre line passes through the centre of each circle. One circle is 3 ft. diam. and the other is 2·25 ft. diam. The distance from centre to centre of the circles is 4·5 ft. and the rectangular piece is 1·2 ft. wide. Find the C.G. of the piece.

3. A rectangular beam is 6 ins. deep by 3 ins. wide. Draw the resistance figure and find the modulus of section. Check your result by calculation and find the safe distributed load on a 4-ft. span if the stress is not to exceed 2500 lbs. per sq. in.

4. Draw the resistance figures for—

(*a*) A square, 4 ins. side, with diagonal vertical,

(*b*) A circular section, 4·5 ins. diam. (solid),

(*c*) A hollow circular section, 8 ins. O.D. and 4 ins. I.D.,

(*d*) A tee section, 1 in. metal throughout, 6 ins. deep and 4 ins. wide, on both bases.

5. A cast-iron beam section is 18 ins. deep. The tension flange is 8 ins. × 3 ins.; the compression flange

6 ins. $\times 1\frac{3}{4}$ in.; and the web $1\frac{3}{4}$ in. thick. Find the C.G. Draw the resistance figure and find the modulus of section.

Calculate the safe distributed load for an 8-ft. span if the stress in tension is not to exceed 2000 lbs. per sq. in.

6. Draw a horizontal line AB, 8 ins. long. Apply forces, P, Q and S, 3 ins., 5 ins., and 8 ins. from A respectively. Find the moment of inertia of the system about the point A when the forces have the following values:—

P	+8 lbs.	+6	+8,
Q	+6	−10	−7,
S	+10	+11	−10.

7. Draw out the rail section shown in Fig. 124 to such a scale that its depth is 8 ins. Find its C.G. Draw the resistance figure and find its modulus of section.

8. Using the section of Question 7, find, by Mohr's method, the moment of inertia of the section, and check your value of z found from the previous question.

9. A reinforced concrete beam is 18 ins. wide and 24 ins. deep to the centre of the reinforcement, which consists of 6 bars, 1 in. diam. Find—

(*a*) Position of N.A.,
(*b*) Value of I (total),
(*c*) Value of z.

Assuming a beam of this section simply supported at the ends, find the maximum safe span for a distributed load of 1 ton per foot-run, if f_c must not exceed 600 lbs. per sq. in.

CHAPTER XII

RETAINING WALLS

A RETAINING wall, generally of masonry, is the name given to such structures as are employed to sustain earth or water pressure, although the expression is used more particularly in connection with earthworks, while in waterworks the term *dam* is more commonly used. The forces acting on retaining walls and the nature of the stresses induced are quantities which, at the present time, are still the subject of much discussion and difference of opinion. The whole subject is a wide and very complicated one, and only problems of a very elementary nature can be treated here.

The most straightforward case with which we have to deal is the retaining wall or dam as used to sustain a water pressure. The arrangement is shown, in its crudest form, in Fig. 130. Obviously the water must exert a pressure, such as P, on the back of the wall, and our knowledge of hydrostatics teaches us that P must act normally to the wall, on the inner face. It will readily be seen that the force, P, will tend to overturn the wall about the point O, the magnitude of the overturning moment being equal to $P \times H$. Now, the weight of the wall, W, acting at a

distance H^1 from O, tends to keep the wall steady by resisting the overturning moment. The condition of balance is obtained when $P \times H$ just equals $W \times H^1$, and the wall would then just be on the point of overturning. In practice, however, this point must never be reached, and, to ensure stability and certain other conditions regarding the stresses at the joints, it becomes necessary to impose certain restrictions.

If we combine the two forces P and W and find their resultant, then the relationship between P and W must be such that the line of action of the resultant must cut the base within the middle third of its width.

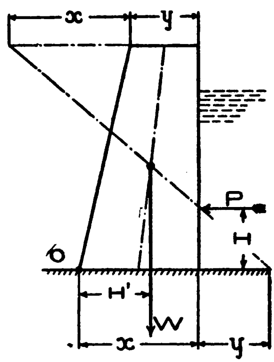


Fig. 130.

Referring to Fig. 131, let ABCD represent a cross-section of the dam. It should be noted that, in working out problems on retaining walls, we always deal with 1-ft. length of the wall. In this case let—

A = cross-sectional area of wall in sq. ft.,
w = weight of material of wall in lbs. per cubic ft.,
h = depth of water in reservoir in ft.

$$\text{Then } W = A \times 1 \times w.$$
$$= Aw \text{ lbs. acting at the C.G. of the section.}$$

As shown by the triangle GHC, the pressure

exerted by the water on the wall gradually increases, from zero at the surface to a maximum at the bottom.

$$\therefore\ P = h \times 1 \times \frac{h}{2} \times 62{\cdot}4.$$

$$= 31{\cdot}2\ h^2 \text{ lbs.}$$

This total pressure acts at a point K, where $CK = \frac{1}{3} \times C.G.$ With the aid of the parallelogram of forces, find the resultant R of P and W.

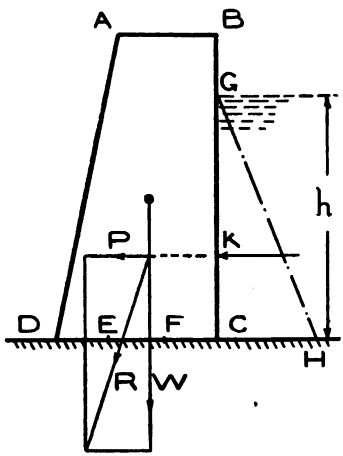


Fig. 131.

Trisect the base DC in E and F, and all the necessary conditions of stability will be fulfilled if the line of action of the resultant (R) cuts the middle third (EF) of the base.

It should, however, be noted that many cases occur in practice where the resultant falls outside the middle third, so that the above rule, although giving ideal conditions, is not of universal application.

Example.—*A masonry dam,* 8 *ft. high, is* 5 *ft. wide at the base and* 1 *ft.* 8 *ins. wide at the top, while the outer face has a batter of* 1 *in* 8. *Under certain conditions the water rises to the top of the wall. Determine graphically if the wall is safe under these*

conditions. Take the weight of masonry as 150 *lbs. per cubic ft.*

$$P=31{\cdot}2\times h^2=31{\cdot}2\times 8\times 8,$$
$$=1995 \text{ lbs.}$$

Note that this force acts normally to the inner face AB.

$$W=\left(\frac{5+1{\cdot}66}{2}\right)\times 8\times 150,$$
$$= 3{\cdot}33\times 1200,$$
$$= 4000 \text{ lbs.}$$

Find the C.G. of the wall section in the point O, and, through O, draw a vertical to represent W. Produce the line of action of P to intersect O*b* in *a*. Mark off *ac* equal to 1995 lbs., to scale, and *ab* equal to 4000 lbs. Complete the parallelogram and join *ad*. It will be seen that R cuts the base outside the middle third, so that the wall may probably not be safe.

The solution of problems dealing with the stability of retaining walls for earth pressure presents more difficulties than the preceding case, chiefly on account of the uncertainty of the magnitude of the thrust which the earth exerts on the back of the wall. Fig. 133 shows a section of a retaining wall, the earth being level on the top and flush with the top of the wall. Earth, like other materials of a similar nature, will be found to assume a certain definite slope when tipped on to a level plane, such as C.G. The slope assumed is shown by the line CE, and the inclination of the line, measured by the angle φ, is termed the *natural slope*, while the angle φ is termed the *angle of repose.*

Obviously, none of the earth to the right of the line CE can have any effect on the wall, since it is able to retain its natural slope without any assistance other than the friction of its own particles. The angle φ is the *angle of friction* for the earth composing the mass, and our knowledge of mechanics teaches us that a particle of earth, lying on this plane, will just be on the point of slipping down. It is only the triangular portion of earth, BCE, which the wall is required to support, and the problem is, to find out what horizontal force this mass of earth exerts on the wall.

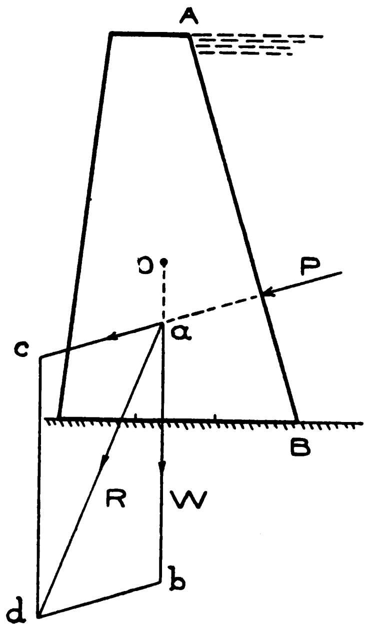


Fig. 132.

Various theories have been advanced towards the solution of this problem, and the theory which for many years has found most favour in this country is that due to Professor Rankine.

Rankine's Theory.—Rankine's theory is based on an examination of the conditions influencing the

equilibrium of the individual particles of earth forming the mass. When the earth surface is level and the back of the wall perpendicular, Rankine states

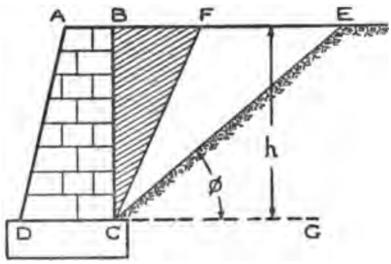


Fig. 133.

that the horizontal pressure can be determined from the following formula:—

Let P = horizontal force acting on the wall per ft. of length in lbs.,
h = height of wall in ft.,
w = weight of earth in lbs. per cubic ft.,
φ = angle of repose.

$$\text{Then } P=\tfrac{1}{2}wh\times\left(\frac{1-\sin\varphi}{1+\sin\varphi}\right).$$

This force acts horizontally on the wall at point $\frac{1}{3}h$ up from the base. It is a well-known fact that this formula gives values of P much in excess of the actual values existing. The difference in many cases amounts to as much as 30 per cent.; it, however, errs on the side of safety, which is a distinct advantage, considering the uncertainty of the problem.

Coulomb's Theory.—This theory is based on the

fact that the earth lying above the natural slope, *i.e.*, the triangular portion BCE (Fig. 133), will tend to slide down the plane and so tend to overturn the wall. It has been shown, however, that the overturning effect of the portion BCE is much less than that due to the smaller triangular portion BCF, and, in fact, that the maximum overturning moment is obtained when BCF tends to slide down a plane CF, which is so situated that CF bisects the angle BCE. This plane is known as the *plane of rupture.* To show the application of this method, we will now consider a simple example.

Example.—*A retaining wall,* 14 *ft. high, is* 5 *ft. wide at the base and* 2 *ft. wide at the top, the back of the wall being perpendicular. This wall is required to sustain the pressure of a bank of damp clay which is flush with the top of the wall. Determine whether the conditions of stability are fulfilled if the angle of repose is* 45°. *Damp clay weighs* 120 *lbs. per cubic ft. and masonry* 170 *lbs. per cubic ft.*

Fig. 134 shows the arrangement, the angle BDK being equal to 45°. Draw DE, the bisector of BDK, and find the C.G. of the triangle BDE, in the point O′. The length BE will be found to measure approximately 5·75 ft. Hence, calling W^1 the weight of the prism, whose section is BDE, we have $W^1 = (\frac{1}{2} \times 5{\cdot}75 \times 14 \times 120)$ lbs. $= 4825$ lbs. Now, find the C.G. of the wall section, ABDC, in the point O, and calculate the weight, W, of 1 ft. length of the wall, thus—

$$W = \left(\frac{2+5}{2}\right) \times 14 \times 170 = 8330 \text{ lbs.}$$

Now, we have first to find the magnitude of P. Coulomb's theory takes no account of any friction between the earth and the back of the wall, hence the force, P, will act normally to back of the wall,

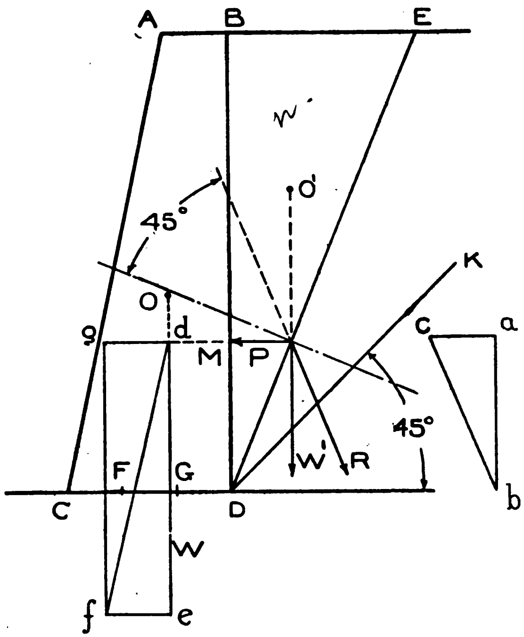


Fig. 134.

and at a point M where DM = $\frac{1}{3}$ DB. The lines of action of W^1 and P intersect on the plane of rupture. Now, if no friction had existed between the mass of earth, BDE, and the plane of rupture, DE, then the reaction, R, would have been normal to DE, but,

since the angle of friction is 45°, we can obtain R's line of action by drawing a line making an angle of 45° with the normal to the plane, as shown. The three forces, W^1, R, and P, are in equilibrium, hence we can determine their individual values by drawing the triangle of forces *abc*. Draw a vertical through O to represent the line of action of W, and produce

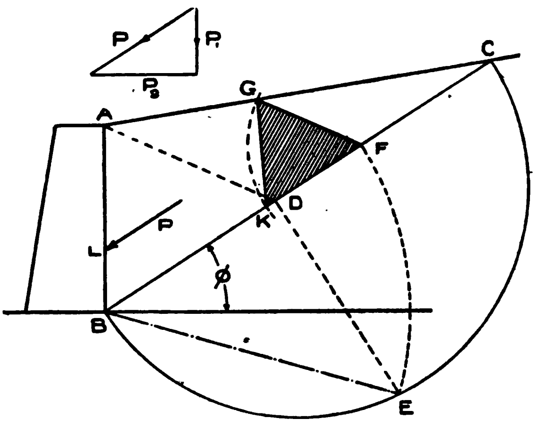


Fig. 135.

the line of action of P to cut it in *d*. Mark off $dg = ac$ and $de = W$, and complete the parallelogram. The resultant will be found to cut the base within the middle third, FG, hence the conditions of stability are fulfilled. For more exact methods, making allowance for earth friction at the back of the wall, the student is advised to consult the various text-books on this subject.

Rebhann's Method.—It might be advantageous to

indicate one graphical construction which has been used for many years by Continental engineers. Rebhann's construction is based on Coulomb's theory, and seems to give results which are quite satisfactory and in agreement with actual practice. Fig. 135 shows a section of a retaining wall, AC being the level of the earth, which, in this case, rises to a point higher than the top of the wall. The natural slope is indicated by the line BC, on which line describe a semicircle, BEC. At A construct an angle $B\hat{A}D$ equal to 2φ, and from D draw DE at right angles to BC. With centre B and radius BE, cut BC in F, and draw FG parallel to AD. Make FK equal to FG and join GK. Then the triangle FGK is termed the *earth pressure triangle.*

Let A = area of FGK in sq. ft.,

w = weight of earth in lbs. per cubic ft.,

P = resultant pressure on wall in lbs. per ft. of length.

Then $P = (A \times 1 \times w)$ lbs.

Mark off BL equal to $\frac{1}{3}$ BA, and through L draw the line of action of P parallel to the natural slope. P and W may now be combined in the usual way with the aid of the parallelogram of forces. P can be split up into its horizontal and vertical components, as shown in the force triangle above.

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